MATHEMATICS AND THE GENERAL PUBLIC

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1. The State of Affairs

In view of the diversity of the contributions sent before the two round tables, Mathematics and the General Public of the first European Congress of Mathematics, it appears that the relationships between mathematics, its developments, the working methods of researchers and the popularization of knowledge constitute a whole set of questions that has more to do with the world of mathematicians than is often thought, and on a more international basis as well.

There is a long tradition of remarkable experiments in Eastern countries, notably in the ex-USSR. As their political regimes crumble one after another, could it be that Western European countries are beginning to be concerned with consciousness-raising campaigns that have proved their worth, especially among youth?

We can acknowledge the Leeds meeting in England as one of the first important conferences on the subject [1]. This meeting was organized by ICMI (International Commission on Mathematical Instruction) from September 17 to September 22, 1989. During that week, academics, journalists, students and educators were able to meet and compare the situation in their respective countries, particularly regarding public appreciation of mathematics and the potential recruitment of future researchers. There were exhibits, competitions, lectures and quite a bit of committee work. Personal experience came as a welcome complement to reflections on communication on various levels. The meeting resulted in a synthesis that was meant to serve as a documentary basis for subsequent work. In the foreword of this document, Geoffrey Howson and Jean-Pierre Kahane wrote: “How do we know that we have a problem?” These words are still significant three years later in an internationally worrisome situation.

There is no doubt about it. In most developed countries the public image of mathematics is bad. Jokes appear in the newspapers; stereotyped, incorrect views about mathematics abound. “All problems are already formulated”... “Mathematics is not creative”... “Mathematics is not a part of human culture”... “The only purpose of mathematics is to sort out students”... “Mathematics may be important to other people, not to me”...
Even when it seems positive, the image is usually wrong: Mathematics is always correct, providing absolute truth, solid and static.

The image of mathematicians is still worse: arrogant, elitist, middle class, eccentric, male social misfits. They lack social antennae, common sense, and a sense of humour.

This is not new. “Mark all mathematical heads which be wholly and only bent on these sciences, how solitary they be themselves, how unfit to live with others, how unapt to serve the world”. This view of mathematicians, expressed by Roger Ascham, 16th century educator and tutor to Queen Elisabeth I of England, is one which is echoed in many later writings. Blaise Pascal, who was himself intimately concerned with mathematics, used to contrast “esprit de géométrie” (a mathematical mind) and “esprit de finesse” (an accurate mind). The latter was an attribute of “honnêtes gens” (nobility and high bourgeoisie), whereas the former was poorly regarded. This contrast has been a favourite theme for dissertations in French high schools, and has contributed to the view of mathematicians as strange characters, divorced from the real world.

It happens that mathematicians often reinforce this view by their behaviour or their writings; there is no shortage of examples, many of them famous. However there are figures of a very different type (think only of Sonia Kovalevskaya) and, of course, most mathematicians do not stand out from the crowd.

People may have very strong feelings about mathematics and their relation to it. Mathematics yields emotional experiences. It can be seen as fun and exciting or as repulsive. It can be threatening, it can lead people to seek security in it or away from it, or it can just be dull. For some people, mathematics is a big blank, or something to be avoided in the future if at all possible.

As a school subject it is needed to pass examinations. It often has high status because of the intellect it seems to require; failure in mathematics may lead to a loss of self-esteem.

G. Howson and J.-P. Kahane also pointed out a few surveys that demonstrated that the public image of mathematics among youth was globally poor. If such is the case, what can we expect of the mathematicians from the next decades? This question was also raised in France during the Mathématiques à venir colloquium that was held in December 1987 at the École Polytechnique and for once received unexpected press coverage [2].

“How do we know that we have a problem?” It was enough to hear what two young girls, just finishing high school, had to say about mathematics during the second session of the round table last July. Their never-ending complaints about the lack of intellectual purpose of mathematics and its total lack of any sensory context provoked a good deal of turmoil among mathematicians who were there. The way these students set mathematics in opposition to philosophy on the grounds that
the latter was more “human” and more connected to our perceptions of the world, incited several reactions. According to Jean Brette, director of the mathematics “department” of the Palais de la Découverte in Paris, “the teaching of philosophy is closer to the actual work of the philosopher than the so-called school mathematics is to the work of the mathematician”.

Jean-Michel Kantor and Philippe Boulanger (Editor of the French journal Pour la science) expressed this lifelessness in a similar fashion in a preparatory text for the European Congress of Mathematics:

What is the image of mathematics in the general public? It cannot be said that it is bad. Rather, it is evanescent. The general public, outside of the professional mathematical milieu, does not have a clue of what contemporary mathematical research is about. Mathematics retains its image as a completed science that is learned without enjoyment in high school. It is a challenge to evoke the cultural aspect of mathematics. It is fashionable to be proud of being ignorant of it, as Bouvard or Pécuchet would have put it: such ignorance is almost a sign of sanity.

The fetishistic example of Bourbakism also gave mathematics the image of a science consisting of abstract structures, in which general synthesis came before examples and formalism concealed meaning.

As early as the 1960s several reactions against this tendency had appeared, some of them related to a change in mathematical research itself. Mathematics sprouted outside of established structures and “abstract algebra” and theories that were more easily illustrated such as fractals or knots came into view. At the same time, computers brought computing aid and the visualization of results to mathematics. There were parallel undertakings to publicize the dynamics of mathematics through films, articles in multidisciplinary popularization magazines, exhibits such as the minimal surfaces exhibit at the Palais de la Découverte or at the Cité des Sciences et de l’Industrie de la Villette, specialized journals for high school students (L’ouvert, Tangente, Quadrature, Maths et Malices, Le nouvel Archimède), activities for youth such as Maths en jeans and competitions such as Kangourou.

These French undertakings nonetheless suffer from a lack of coordination, from the regrettable absence of mathematics in the most widespread medium, television, and from a lack of interest on the part of the mathematical community, which has better things to do, or so it seems. The torch brilliantly held up by Poincaré and Borel was only taken up by Ivar Ekeland, Marcel Berger and few others. Popularization is more alive in Russia and the Anglo-Saxon countries.

Hasn’t the time come to eliminate the sophism that thinks that information delivered to the public never deals with mathematics because mathematics is incomprehensible and that this silence of the media endlessly reinforces this incomprehensibility?

A consequence of this is found in the Frankfurter Allgemeine Zeitung by Bernhelm Boos-Bavnbek and Philip Davis:

Given the internal difficulties, it is not surprising, that, generally speaking, the media avoids mathematics. As a measure of this avoidance, consider a few statistics. The Frankfurter Allgemeine Zeitung, considered one of the most intellectual newspapers in Europe, has a weekly science section and runs perhaps 20 stories per issue. An informal scan of 225 such stories revealed that not one of
them was related to mathematics as such. If mathematics underlies a statement, it is left hidden, or at best is sloughed off with the sentence “a mathematical model shows that...”.

2. Obstacles

The most common reaction is that of the two Parisian high school students. Media people and a good part of the public have the same reaction, although books and conferences capture their curiosity more than they used to. Everybody has the impression of understanding the main evolutions of astrophysics or medicine or the impact of progress in biotechnology. Ethical or civic problems that arise from the use of biotechnology rarely meet with indifference. But in mathematics how can one be anything other than a passive witness to an incomprehensible discourse?

Bernhelm Boos-Bavnbek and Philip Davis go even further in referring to intrinsic characteristics of the discipline:

Another difficulty of the communication of mathematics is that people have fundamentally different concepts of clarity or lucidity. While the mathematical community is aware that it is tongue-tied with respect to the general public and often works hard to achieve reformulations that have a higher degree of popular comprehensibility (in some instances, it has taken centuries to do this), the fact remains that the professional mathematician has a concept of lucidity that is different from that of the nonprofessional. To the mathematician, what is lucid is that which proceeds by logical or computational steps from sharply formulated assumptions to conclusions. This procedure supports an awareness of the intricacies of human thought and allows a dramatically growing complexity in its treatment of reality. Its explication demands a kind of patience, concentration and attention to detail that is poison in a media presentation.

The public wants a story; lucidity is located in the narrative mode. The story should be simple, understandable, definite, resolved, interesting. And in today’s world, interesting often means sensational. A story should tell of a great accomplishment or of a great failure, should have a hero or a villain, should carry some kind of emotional charge. The public wants the concrete; it does not like the abstract, and mathematics is surely one of the most abstract of the human constructions. These considerations rule out most mathematical developments and applications.

Pursuing their argument, the two authors ask: “What is mathematics?”

What is mathematics? Mathematics is the science and the craft of quantity, space and pattern. The first two are reasonably clear, but by pattern is meant not only visual pattern, but also any mode or organization that is distinguished or imposed and occurs repeatedly. A short definition such as this is inadequate to convey the
full extent and richness of the field. One might be inclined to add that it is the
deductive study of these three notions and of the symbolisms through which this
study is pursued.
Every special field from horseracing to banking to medicine has its special terms;
likewise mathematics, and the special terms and ideas of mathematics are stan-
dardized, abbreviated and written with special typographical symbols. The ten
digits themselves, the mathematical signs plus, minus, etc., the square root sign
are met in the elementary grades. In advanced mathematics there are literally
hundreds and hundreds of special signs and symbols employed, hundreds and
hundreds of ordinary words given special and precise meaning, and this mean-
ing is often located only in combinations of previously agreed upon terms and
symbols. Thus mathematics is like a language, with some features of a natural
language such as English or Swahili, but with many differences. The general
public is at a loss as to how to pronounce the complicated formulas that often oc-
cur in mathematical statements. This is the least of the difficulties. English may
be translated into Swahili and vice versa, granting that certain subleties may be
lost either way, but it has become increasingly difficult to translate mathematical
statements into the English of ordinary experience even when the mathematics
itself is expressed in what seems to be English.

Take the following mathematical sentence (theorem) : “The zeros of successive
real orthogonal polynomials separate one another”, which occurs in an area of
advanced mathematics with which one of the authors is familiar. If a lay person
asked him for the meaning of this sentence, he might reply that certain curves
interlace. He might even draw a picture. But he would be hard pressed in lay
terms to state precisely what curves, how the interlacing occurs, how he knows
this is to be the case, what the connection is between this mathematical sentence
and others, or why anyone should trouble his head over the whole matter.

Mathematical terms, signs, formulas have a precision and an abstraction lacking
in ordinary language. They usually point to themselves. This lies at the heart of
the difficulty. Non-mathematical signs, even difficult ones, point to something in
the world of physical existence or event. The meaning of mathematical signs is
usually to be found only in the way in which they are used in accordance with
some conventions of our brain.

Scientific signs are bound to one scientific context : “Hepatitis B”, “Amorphous
semi-conductors”, “Ozone O₃”. Such terms are media-friendly. Mathematical
signs, on the contrary, being abstract, are movable from one context to another.
They can tell too many stories at one time. The stories are too unspecific. They
often lack a visual or kinesthetic interpretation, which, if present, makes for easy
understanding. They make no immediate appeal to our allegorical or mythical
natures as do the statements of the biological, medical, physical, astronomical
sciences. Accordingly, they are not media friendly.
According to Wittgenstein (and to every kindergarten teacher as well), showing and telling are independent modes of communicating knowledge. But there is also a kind of mathematical knowledge, located in the comprehension of the way its component parts fit together and operate, that transcends these standard modes and is not easily communicated.

Mathematics, when it is applied to the physical or social worlds, stands “infra” to some topic of more primary interest to the general populace. When mathematics is self-referent, it often displays a self-indulgent triviality of motive that can baffle all but the most passionate of its votaries. Perhaps it is the fate of all infrastructures, whether of language, of finance, of technological availability, to be taken for granted or ignored. What captures attention is the superstructure; like electricity in a blackout, what is “infra” is noticed only in a crisis. As Tolstoy once put it, “The leaves of a tree delight us more than the roots.”

Still, is the question itself legitimate? According to Catherine Goldstein, “To choose to formulate the entire problem of the popularization of mathematics in terms of the ‘definition of the subject’ is to accept in advance a referential framework devised for privileged westerners formed in a literary culture”.

We must add to this debate the even more provoking contribution by Marie-Jeanne Husset, a scientific journalist, chairperson of the second session of the round table, “Do we really need to popularize mathematics? Or can people do without an area of knowledge that is difficult and abstract?”

Will these obstacles constitute a hindrance, when some members of the mathematical communities have already engaged in a reflection on the means needed to facilitate relations with their scientific colleagues or simply with their fellow citizens?

3. Assets

In effect, every obstacle reveals new reasons and new advantages for the popularization of mathematics. In particular, the object of mathematics is not to exploit an area of the physical or social world but to devise non specific concepts and general methods. The value of mathematics lies in its availability and accessibility to the broadest and most varied public possible through education, documents or personal conversations.

There exist several documents which, upon careful reading, reveal a few key difficulties and needs one encounters outside of the mathematical community. One of these is the 1989 report on circumstances of the Centre National de la Recherche Scientifique (CNRS) [3]:

Pasteur thought that there is no good applied science without a good theory. Although a theory does not necessarily lend itself to a mathematical formulation (cf. in contrast to
the current trend of using the vocabulary and some of the concepts of biology in computer science), many theories in fact allow it. In order to be able to use such a formulation, it is necessary to identify the obstacles that may arise.

If mathematics has returned in recent years to more open relationships with other sciences, this has not always been the case. H. Weyl (1919) is supposed to have said that mathematics and physics are like a couple who quarrel in the daytime and nourish each other in the dark of the night. According to this view, physicists dream of a mathematical supermarket where all the tools they need are available when they need them, and mathematicians get exasperated or even irritated when their concepts and tools are ignored or badly used.

This is by no means the end of the list of difficulties that arise in the interaction with other sciences. In fact, there are numerous mathematical problems coming from other mathematical problems, themselves coming from other sciences, that are currently unresolved due to lack of proper tools. Actually, mathematicians are not always able to answer the questions that are asked at the time they are posed in a satisfactory fashion.

There are several reasons for this situation. Let us assume that a mathematical model has been built to describe a phenomenon but that the analysis of this model leads to problems that are out of the reach of contemporary mathematical techniques. If mathematicians deem such a problem interesting, they take hold of it, expose its structures and isolate the difficulties and, eventually, solve it. However, the solution may require an amount of time that has no common measure with the delays that are customarily accepted by the end users. In effect a theorem is always true and a mathematical concept always useful, even 2000 years after its discovery or proof. This is rarely the case in other sciences and certainly never in the economic world. Nonetheless, it often happens that advances in mathematics made for the solution of this problem can help solve other problems.

It may also happen that, because the relevant parameters and fundamental interactions of a phenomenon fail to be identified, no reasonable and efficient model can yet be obtained. This is often still the case in biology and economics. However, even in such a case mathematics is useful in pointing out the absurdity or the limits of accepted models because it can be used to derive all their consequences.

In the meantime, the field of interaction has considerably broadened, but communication difficulties are still numerous. Results that are obtained in different languages and with different problematics often require years of mutual adaptation in order to be understood by both sides until they finally merge into an embryo of common culture. For a given theory, the important concepts for the mathematician are often not the same as for the other specialist, hence the difficulty in accepting the other's point of view and reading his publications. It is thus necessary to compile "dictionaries" to convince everyone that they are actually talking about the same concept (a recent example of this is given by the various approaches to the renormalization group in physics and in mathematics).

Since the number of professional mathematicians is limited and since their ability to grasp radically new fields is somewhat held up by the demand of the mathematical milieu to publish only really profound results, every new area of interaction almost always necessitates a new generation of mathematicians. Given the current organization of their community, mathematicians thus tend to react rather slowly.

Interaction is not encouraged by the educational system, especially in secondary education. For example, it is regrettable that high school teachers do not have more contacts with the utilizations of mathematics, due notably to the lack of appropriate documentation.
A possibly more irremediable hindrance to interaction comes from different aims and frames of mind. Mathematicians are happy with the harmonious unfolding of a theory. Due to their training, they are less sensitive to the notion of relevant parameter and to the limits of modeling. Mathematical theories have a very long life span—mathematics is somehow timeless. Mathematicians are perfectionists and are more likely to appreciate the difficulty of a given result than its usefulness. An elegant new proof of an old result is well appreciated by mathematicians since it shows a deeper understanding of the nature of the result. The distinction between pure and applied mathematics then is more a question of frame of mind than of the actual applicability of the results or of the qualification of the tools used. Applied mathematicians are in effect forced to include a dimension of utility in their work, which can lead them to reorient their research direction if their mathematical field is no longer in touch with a potential domain of application.

The same report quotes several areas where obstacles to dialogue have been overcome and research has benefitted from interaction between mathematics and other sciences. As we have already noted, mathematics does not have a privileged grasp on reality, but it is versatile enough that it can become involved in a large variety of domains. Coming back to the report, one interaction mode is rapidly gaining importance—modeling.

Mathematical modeling is not a recent creation. However, due to the current computing capabilities of computers, it is being applied to new areas. Because modeling gives structure to the phenomena under study, it is an efficient tool in pointing out causality effects and significant parameters. Furthermore, by suggesting similarities between isomorphic models, modeling can help transfer intuitions and problematics from one domain to another domain (for example, the recently discovered surprising similarities between arithmetic geometry and quantum field theory). Due to internal classifications that are made available through mathematical modeling, a certain number of qualitative behaviors can be sensed \textit{a priori} even though current mathematical techniques are insufficient to completely solve the resulting problems (\textit{e.g.} all the models that pertain to a given class of differential or partial differential equations, such as equations of elliptic, hyperbolic or parabolic type, when specialists in classical mechanics are led to use equations whose type may vary throughout the domain). Mathematical modeling can also, at least in the best of cases, be a great thought-saving device by providing systematic procedures for studying and solving problems.

However, mathematical modeling can only make use of the available tools. Models can be linear or nonlinear, discrete or continuous, in one or several variables, stochastic or deterministic, stationary or time-dependent, and so on. The mathematical background of the mathematician who does the modeling is often the determining factor in choosing the nature of the model. This fact can become more important than the phenomenon itself. For example, the recent advances in probability theory and their broad diffusion can be seen to result in more and more models that take chance into account. Good modeling therefore requires very experienced mathematicians who are able to overcome the technical problems that arise during the construction of their model.

In any case, if mathematics can help in understanding a phenomenon, it cannot under any circumstance be a substitute for reflection on the phenomenon and on the relevance of the model. A model only contains what has been put into it. The role of mathematicians is to extract every bit of information from the model. In doing so, mathematicians may have to make accessible information from other disciplines by using similarities that the
mathematical processing of the model may suggest.

It is certain that the use of mathematics can become a danger when its only role is to throw dust in your eyes by concealing a lack of thinking concerning the phenomenon under study through a brilliant mathematical development.

A good model and a good theory must be predictive and must be able to interface with experimental reality..., even if modeling is sometimes used in those very situations where experiments are difficult or even impossible to realize (space and nuclear industries, economics, medicine and so on).

According to these remarks, if defining mathematics is still difficult, explaining its areas of applications is comparatively easy. For example, differential equations and dynamical systems are related to the description of the evolution of phenomena, probability theory to deriving certainty from uncertainty, harmonic analysis to decomposition and reconstruction by means of elementary vibrations. Mathematical concepts that arose from telecommunications such as information theory or coding, are now influencing molecular biology. The current vogue of wavelets is due to the fact that wavelets provide a unifying and generalizing framework for a host of preexisting engineering, physical as well as mathematical practices, making them at the same time more powerful and more easily communicable. Finally, what can we say about fractals except that one meets them everywhere? It thus appears that while mathematics is not just a discipline at the service of other disciplines, it is also not disconnected from other sciences. It is the very generality of mathematics that makes it necessary to integrate it into the common cultural patrimony.

There is thus not one unique way of engaging in mathematics, but a multiplicity of ways, as demonstrated in several recently published works (see for example the book by Mauduit and Tchamitchian, which is aimed at a young readership, [4]).

This multiplicity is based on the variety of subjects and their applications and also on the variety of conceptions of what it means to do mathematics.

A universal language, an intellectual game, sport or exercise? After Miguel de Guzman ("Mathematics is a generator of true beauty, intellectual beauty, profound beauty"), Jacques Mandelbrojt, a physicist and an artist, draws a parallel between art and mathematics by observing as a jest that you more often hear a mathematician speaking of a beautiful theorem than an artist speaking of a beautiful painting. Mandelbrojt also notes the analogy between the role played by mathematics in physics and the role played by abstract painting in relationship with figurative painting. To link the mathematician’s methodology to that of other scientists, to make the “interactions” understood, as the CNRS report puts it, are undertakings that can reach beyond the limits of the scientific community and concern a broader public.

Lacking a global view of the profusion of mathematics, we can try and make use of the variety of access that will all shed a different light on mathematics. We must allow every subject, use all media and reach the entire public, and once more
transform all obstacles into assets. It is remarkable that the mathematicians who were interviewed before the Congress placed their trust in the written word, which is the traditional way of transmitting abstract knowledge, on the one hand and on television on the other, which provides a sort of acknowledgment of the credibility of any information or know-how that is now necessary. It is as though there were no intermediate public between the colleagues who are determined to read a paper from beginning to end, even if it is slightly technical, and a vague population for whom the transmission of knowledge has to be mediated through images, through a few documents summarily shown on the screen between images of some conflict somewhere on the planet and the last electoral speeches.

On the contrary, according to Catherine Goldstein, "popularization can be a practice as well and not only diffusion of knowledge. There are several undertakings in this direction: Maths en Jeans in France tries to initiate high school students to research; on a completely different scale Paulus Gerdes in Mozambique demonstrates the mathematics necessary for agricultural techniques. More generally, ethno-mathematicians try to build on those practices of mathematics that can be put to work based on the culture of a given group—like typography, weaving, knitting or cooking". Besides, several films, radio programmes or competitions such as Kangourou in France have widened the spectrum of popularization activities. We will come back to this in the next section.

The discussion on the trends, what is at stake and even the nature of mathematics is also, and even primarily, a popularization means. It concerns all levels and the entire public. The question is not to teach but to bring awareness to an area of human activity. Let us again give a few examples. A debate on “Mathematical thought and reality” held the attention of a non scientific audience for a whole afternoon at the University of Orsay. The people who buy (and who read) books signed by Changeux and Connes or by Penrose are also primarily interested in the philosophical aspect, the nature of mathematical objects or the distinctions between thinking man and a sophisticated software. Once the public’s attitude has been changed, new areas of interest may open up. There is no doubt that the discussion on mathematics and philosophy also belongs to the popularization of mathematics.

Of course, all these ways of approaching mathematics should not be thought of as equal. A meeting between youth and professional researchers or a lecture with slides and overhead projector will not have the same impact and will not command the same attention as a radio or television production of a few anecdotes on the somewhat chaotic adventures of nonstandard analysis. Portraits, which present the advantage of making the profile of a professional mathematician more familiar and in principle less caricatural, are more catchy when traits are exaggerated. Here is another obstacle, and potential asset, namely, the general public’s ignorance of contemporary mathematicians, at least in some countries like France.
The rigidity of school mathematics is often opposed to any reflection on raising consciousness concerning great mathematical problems. School mathematics is more a discipline than a science, more a finished and linear product than a field of open questions. Its discriminatory role in curricula cannot be dismissed either. However, we must not underestimate the classical aspects of mathematics by replacing them with an immersion in which showing how to do it replaces actually doing it. On the other hand, mathematics teachers have an enormous potential for diffusion and communication. They can be involved in journals, magazines, brainstorming sessions and all sorts of competitions. It is out of the question to confine mathematics teachers to the lowly task of evaluating ossified knowledge. They stand on the front line of the diffusion of mathematical culture. Finally there exists a long tradition of great teaching treatises starting with Euclid up to Bourbaki that recalls this privileged link between research and the transmission of its results.

To sum up the spirit in which mathematicians should approach “efficient popularization”, we first quote a general principle of Miguel de Guzman:

To pay attention to the place where the persons are to which we want to pass on our message and to examine what society and the general public are ready to receive.

We then take the “recipes” proposed by Philippe Boulanger and Jean-Michel Kantor in a humorous fashion in the document presented to the Congress:

“Popularization requires efforts from the mathematical community and cannot be left to journalists, editors and producers by themselves. The latter, with the help of the mathematical community, could incite some of its members to take up the pen. Such an ideal cooperation has not yet proven very efficient.

In order for mathematics to be interesting, it has to be:
(a) appetizing,
(b) palatable,
(c) and profitable for the persons making the effort to swallow it.

(a) Appetizing mathematics

The primary cause of the lack of appetite for mathematics is the vocabulary, semantics and careful language that rigor requires. An elementary precaution should be to insist on rigor of vocabulary only when such affectation is important for the specific problem under consideration.

1. The subject is more appetizing if one starts from an example that is at the same time striking and as simple as possible in order to get to the heart of the subject. An example is interesting when it is unexpected, either if the result could not be presumed in advance, or if it has to do with the mathematization of an ordinary problem.
2. An historical approach can embark the reader or editor on a stream of thought or a traditional form of research in which new results have recently been obtained.

3. A mathematical result that has applications in other sciences or in solving concrete problems has more impact.

4. Game theory applied to simple games stimulates the reader’s analytical faculties and gives him or her a feeling of confidence if the game is complex enough. Games that are expressly constructed to be mathematizable are not very interesting.

5. Advances in computer graphics prick curiosity and arouse the sense of beauty. They can incite readers to redo on their own computers what was shown to them and get them interested in the theory, e.g. cellular automata.

6. Using computers can be a means of avoiding tedious or even unending computations that in the past were a cause of repulsion. Computers are also helpful in handling mathematical objects and in mastering their study.

7. The purity and truth of a mathematical result detach the reader from a dull day-to-day life.

8. Readers like novelties. They like to feel the movement of ideas. With an appropriate effort, they can understand the evolution of modern research.

(b) Palatable mathematics

1. There is a broad discussion on the use of formulas in texts for the popularization of mathematics. Formulas have the drawback of greatly slowing down reading speed. They also frighten readers who are afraid they might not grasp the entire meaning of the symbolism. However, a formula can have an intrinsic beauty. In this case it can be gently introduced, if such is the goal of the presentation. We must remember that letters can also cause apprehension as they recall school memories while numbers are reassuring by their self-evidence.

2. The mental gymnastics that have to be performed in order to solve a problem can be explained. It is the author’s responsibility to make clear what a proof is and how it differs from a mere presentation. The author must also indicate why such and such a method of proof was chosen (by contradiction, by induction, by analogy with a geometrical problem and so on).

3. The reading speed for a mathematical text must not be too different from that of any other text. It is up to the author to smooth out the difficulties.

4. It is important to relate the elements of the action (i.e., here the proof) with points that are known to the reader. The reader feels more secure if handholding from his familiar universe is available.

5. The classical presentation is not the only possible way to explain a mathematical result. The author can use any form of dialogue and storytelling, all the resources of paradox and drama.

6. The reader must be made to understand that mathematical subjects cannot be
grasped in one fell swoop, but by successive approximations and only exceptionally through flashes of global understanding.

7. Mathematics is an activity and popularizers must be concerned with making the reader a participant.

(c) Beneficial mathematics

1. Is that which lingers in the memory as a pleasurable moment.
2. Is mathematics that is usable in other sciences or that sheds light on some social activities. Probability theory, electoral mathematics, the laws of chance and optimization principles are very much appreciated in this respect.
3. Just like other arts, mathematics is a means of communication within social or familial groups.
4. Mathematical culture helps in mastering the omnipresence of computers. Geometry enriches the mechanistic view of the world and logic and probability theory enrich the analysis of decision making processes.
5. Mathematics can be rejuvenating for those who were familiar with it in their youth.

It would be interesting if an exchange between the various attempts at the popularization of mathematics led to more professionalism on the part of popularizers and to more interaction between mathematicians and media professionals. This can only happen if mathematicians and popularizers feel that they are working together.

The important point is that the author must not despise his public: the art of popularization is difficult and strewn with traps. There are however remarkable achievements such as the books by Penrose, Ekeland or Mandelbrot. These are not easy books and their qualities may deserve further analysis.

Mathematics helps creativity by developing a certain kind of intuition and an aptitude for problem solving. Mathematics does not claim to explain everything (no mathematician ever forced Lacan to explain the unconscious with knot theory). Mathematics, the better one knows it, however helps in discerning the moving boundaries of knowledge. It thus seems important to popularize mathematics if only to assess its power and current limits. But are all means equally good?"

The above text gives some possibilities. Let us stress one more aspect—public expectations. The public is often ahead of the media and professionals in its desire for mathematical knowledge or activities. An example, in the Sciences and Industry Museum in Chicago the mathematics room is very poor in every respect. However, when you leave the museum to enter the museum’s bookstore, half of the books that are on sale and three quarters of the objects and games have a mathematical content. The public is ready to pay for mathematics.

Mathematics has political, economic, social and cultural stakes which can appeal to various elements of the public. For example, the history of mathe-
Evolution, in relationship to the history of ideas, answers a widespread need. The presentation of its recent evolution, as in the CNRS report quoted at length above, can capture the attention of persons who might otherwise be repelled by detailed reasoning. Mathematics is justly considered to be general and powerful and is unjustly considered to be inhuman. Given its importance in today’s world, the aim of popularization is to correct the injustice of this vision and to ensure that no one facing mathematics ever again is left with the impression of facing a closed door marked “no trespassing”.

4. Evolution

Let us take as our starting point the Leeds colloquium of September 1989. As we have already said, the emphasis was on the problems, the bad image of mathematics and mathematicians, their lack of experience at the time and the intrinsic difficulties in popularizing mathematics. However, some directions were already indicated: mathematics in various countries and cultures (Czechoslovakia, Hungary, China), possible subjects, mathematical problems and games, the various levels and the targets (public at large, youth, teachers, professionals, retirees), the role of exhibits and various media (newspapers, television, radio), the transformation of what is “new” into “news” and the responsibility of mathematical societies. Starting from one of the main goals of popularization, which is to change the public’s attitude, the colloquium led to the working hypothesis that the change of attitude of the public would open a new field and new needs in mathematical popularization.

The evolution since 1989 seems to confirm this hypothesis. There is a flourishing of initiatives in Europe and throughout the world that seem to meet the expectations of part of the public.

Let us first recall the importance of what the British called in Leeds “the event”, the “pop.maths.roadshow”. This event was made up of several exhibits (posters, montages, experiments, on-screen graphics, games, sculptures) which drew some 20,000 visitors to Leeds. There was also a series of films and audio or video cassettes as well as popular lectures for youth of different age groups, with such different speakers as Johnny Ball (a British TV star), Christopher Zeeman and Dr. Ruth Lawrence, then age 17. On the whole 20 lectures were given during the week in packed enthusiastic amphitheaters. Another main feature of the event was the conclusion of the “Leeds competition” between high schools, each team working on a real research project (three subjects on coding, tilings and dynamics) and presenting their results on posters, software or film. Add to this the excellent recreative practical exercises such as the math.night club “chez Angélique”.

This event had a measurable impact in Great Britain. The roadshow, in a reduced form of exhibits, toured 21 cities in Great Britain and Ireland between September 1989 and October 1990. The total number of visitors was 250,000.
Geoffrey Wain from the University of Leeds wrote a report for round table participants (“Reflections on the Pop Maths Roadshow”) that was distributed as part of the “Situation in Europe” brochure. This report contains an exhaustive description and some conclusions of general interest. We quote the following one:

... if popularization is to be taken seriously, performances need to be developed that convey to lay audiences the essence of mathematics without requiring of them knowledge of the technicalities. There is an attitude problem here. Many mathematicians may suspect that communication of this kind is demeaning, in some sense, of mathematics itself. The evidence of the Roadshow suggests that the opposite is the case; that many people, when they come to appreciate some aspect of the subject, may well gain a new respect for it and interest in it. It must be likely that, if people appreciate and like the subject at the popular level, more of them will want to study it for its own sake. It is important though that any study they pursue does not destroy their enthusiasm.

Leeds also had a considerable, if difficult to ascertain, impact on the persons who attended the meeting due to its exemplary impression. Let us now review the recent salient initiatives in a few European countries.

In France, following the book by Changeux and Connes already mentioned, the years 1990–1992 saw the appearance of several good books that popularize mathematics. Let us mention the beautiful one by Christian Mauduit and Philippe Tchamitchian, *Mathématiques*, aimed at high school students but touching many extracurricular subjects; the saga by Ivar Ekeland, *Au hasard*, for which its author was awarded the d’Alembert prize of the Société Mathématique de France; the very personal book by David Ruelle, *Le hasard et le chaos*, mixing the mathematical and physical aspects of deterministic chaos; *Géométrie*, by Marcel Berger, at the same time teaching treatise and popularization for the informed audience; *Le pouvoir des mathématiques*, by Moshe Flato, for the cultivated reader. In 1991 and 1992, the *Palais de la Découverte*, whose director Michel Demazure is a mathematician, held two exhibits on mathematical or physico-mathematical subjects, fractals and minimal surfaces. Furthermore, there was a festival of mathematical films there in July 1992. Mathematicians and mathematical societies have increased their communication efforts since the *Mathématiques à venir* colloquium in 1987 and the “Mathecom” exhibition in 1989. Let us also mention between September 1991 and May 1992 the Cerisy-la-Salle colloquium *Mathématiques et art*, the beginnings of an afternoon in Orsay with Alain Connes and Pierre Cartier on “mathematical thought and reality”, the one day meeting *Mathématiques indiscrètes* in Marseille (exhibit, films, debates, music), the special meeting with journalists of the Société Mathématique de France on the theme “mathematics as an historical adventure”, and interesting personal initiatives such as that of Josette Adda at the center for scientific information in Boulogne-sur-Mer or the creation
of the summer mathematics university by Daniel Loeb at Bordeaux. TV films and radio programmes have also been produced in the same period.

Following the Mathématiques à venir colloquium and the Cinquante lycées ("fifty high schools") undertaking, a survey on the image of mathematics among high school students compiled by the Strasbourg IREM (Research Institute on Mathematical Education) in 1989, to which the Leeds colloquium had already alluded, initiatives reaching out to youth have multiplied. The most important ones are Math en Jeans, Kangourou and the Congrès Mathématique Junior (La Villette, July 1992). The principle of Math en Jeans is to have two teams from two high schools work on a given subject with a researcher for several months. The resulting presentations (Palais de la Découverte, April 1992) were reminiscent of the Leeds competition. Kangourou is an adaptation of the Australian mathematical competition that has 500,000 competitors a year. Its success was rapid: more than 100,000 competitors in 1991, more than 200,000 in 1992, with participants from several European countries. Finally, the Congrès Mathématique Junior held during ECM was a successful meeting between motivated youth and mathematicians. France-Culture radio station gave it some coverage and Alexandre Sossinsky, ex-editor of Kvant, gave a very appreciated report of it during the second session of the round table.

The questionnaire prepared by J.-M. Kantor and sent to several European mathematical societies showed the diversity and importance of recent initiatives in most European countries. Here is a small sample of the information received, see the appendices for more details.

In Germany, the centennial of the Deutsche Mathematiker Vereinigung was marked by a big exhibit in Bremen, Mathematik sehen, including, in particular, examples of dynamical systems and deterministic chaos, which was visited by 30 classrooms and was quoted in 25 newspaper articles.

In Norway, the 150th anniversary in 1992 of the birth of Sophus Lie was the occasion for an exhibit on his life and work and for the production of a “Sophus Lie” video, on which mathematicians and video professionals collaborated.

In Spain, there is a vigorous increase in publications in and around mathematics. Particularly noticeable is Miguel de Guzmán’s last work Para pensar mejor [5].

In Sweden, there is a programme directed by Dan Laksov to collect mathematical problems that can be proposed as new activities in Swedish high schools (ages 15–18). A large number of Swedish mathematicians participated in this programme. The resulting book is currently under testing, with good reactions. Discrete mathematics (combinatorics, coding and so on) is strongly represented, together with other subjects (iterations, electoral theory, Möbius transformations, Bernoulli polynomials, Gauss integers, history of mathematics, etc.)
In Czechoslovakia, in the framework of the small, one volume, “general encyclopaedia” prepared by the Academy of Sciences, a new conception of mathematical articles has been proposed and accepted. Aside from classical subjects (circle, sine, gradient) there will also be articles on algebraic topology, Galois theory, cardinals and ordinals, the monster group, exotic spaces in dimension 4 and so on.

We finish this quick glance with information received during or after the Congress.

In Denmark, a volume called *Udsagn* (statements) was published in 1992 on the occasion of the 60th anniversary of the foundation of the association of mathematics teachers. Writers, composers, journalists, engineers, politicians, scientists and mathematicians express their feelings regarding mathematics. The book, illustrated with beautiful photographs, is a great success [6].

In Italy, Michèle Emmer, who was already well-known for his films and popularization articles, has just published three books, predominantly geometrical, [7] [8] [9], and a sumptious special issue of the arts and culture review *Leonardo* on visual mathematics [10]. He declared at the round table:

Until a few years ago it was extremely rare to read articles on mathematics in newspapers, at least in Italian newspapers, or to see mathematical books aimed at the general public. Now the situation has changed. A field has opened in the publishing business for the publishing of such books.

The “Arts and Mathematics” theme that was discussed at the Cerisy-la-Salle colloquium of 1991 [11], was also discussed in Spain in a series of brief surveys on the cultural and social dimension of mathematics aimed at students and teachers, notably through lectures by Capri Corralès (University of Madrid) interweaving pictural developments (from Pompeii to the cubists) and mathematics (perspective and conceptions of space).

We can assess the popularization and teaching of mathematics in Poland in the special issue of “Mathematics” published on the occasion of the 7th international congress of mathematics teaching in Québec (August 1992). “The goal of the journal is to improve the professional knowledge of teachers, both mathematical and educational. We publish articles on mathematics and on mathematicians, and especially articles popularizing mathematics and its history”, says the editorial. A good example of this is an article that starts from a proof of the irrationality of $\sqrt{2}$ to explain what a formalized mathematical theory is.

There was an exhibit *Mathematik in der industriellen Praxis* shown during the Hannover fair (April 1–8, 1992) that was prepared by the Center for Arts and Science of North Westphalia (Wissenschaftszentrum Nordrhein-Westfalen). The exhibit met with remarkable success, witnessed by almost 60 pages of press coverage. A few titles from these articles were *Mathematik ist überall!, Mathematik*
Dangers and Recommendations

It seems that a goodly number of mathematicians have accepted that the concepts of culture and patrimony should include a certain amount of mathematical knowledge. Such knowledge does not have to be sophisticated but it must be linked to the development, history and impact of mathematics. All responses to the questionnaire demonstrated, if unequally, the desire that at least an awareness on the part of the citizen (of the honest man) be achieved.

This is not simple and mathematicians cannot escape from this task. Besides, some of them do not want to leave the job to another intermediary: lack of confidence or timidity in expressing themselves?

If we agree that mathematics and the general public are bound to grow closer to each other, we should nonetheless outline a few recommendations in order to avoid certain pitfalls.

After the dangers caused by the gap between mathematics and the general public, we now have to face the dangers of intimacy, to which neither mathematicians nor the media are well prepared.

1. The spontaneous personal attitude of most mathematicians is adverse to communication. Mathematicians are often simultaneously extremely proud of their discipline and modest and timid concerning themselves. We have to overcome this mix of reserve and arrogance and learn how to talk, in a different context than teaching, to write and above all to listen. There are some suggestions on this in part 3.

2. Most mathematicians suffer from narrowness. They have only a fragmentary view of mathematics and, a fortiori, of its history, its applications and its relationship with human society. Even the great can suffer from this defect. In fact, concern for popularization is the best cure against narrowness. To be able to
extract the important points, you have to question many prejudices and habits. You have to broaden your own views on mathematics and its role in science, history and society.

3. It is tempting to follow the trend and inevitably popularization no doubt intensifies the effects of the trend. This is not too important if one is aware and watches out, especially of course in research, that dominant trends do not smother those to come. The dangers are distortion in the choice of subjects and even more the star system in the choice of speakers. How can we give an image of mathematics and mathematicians that is at the same time truthful and interesting, global and detailed? We must surely take advantage of the variety of research, which is wider in mathematics than in heavier and more targeted disciplines.

4. The profession of mathematics is little known and difficult to describe. This however is not a reason to seek shelter in incommunicability or exotism. We need a large number of young mathematicians for the future and we must be wary that our profession should appear to be at the same time open, dignified and accessible.

5. Scientific and educational deontologies evolved through time. In mathematics, we have a limited experience in popularization compared with, for example, astronomers and the ethics of the relationship between mathematics, media and the public remains to be defined. We must expect tensions and frictions among us.

6. Let us repeat that the media are ill-prepared for mathematics and mathematicians. More precisely, media directors are reticent. We need to work on them. We postulate that this rapprochement is going happen and that the problem is not \textit{are we going to popularize mathematics?} but rather, \textit{à la Polya, what are we going to popularize? Why? How? With whom?}

After having stated the dangers and problems, let us venture into some recommendations.

1. The International Mathematical Union has decided to make the year 2000 an international mathematics year. This decision can be accompanied by big regional initiatives, more media awareness, production of books and films, exhibits and public debates. According to Michele Emmer:

   Even if the period is still far away in the future, we should already start to think about a great exhibition of European mathematics for the year 2000,...

2. Specific actions for youth are possible. This is one of the lessons of the Junior Mathematical Congress. Such actions are necessary in order that the cultural aspects of mathematics become familiar to youth and also in order that young people find the mathematical profession interesting. It would be a good idea to take a census of such initiatives and to stimulate them on the European level.
3. We have mentioned in passing the special and indispensable role of mathematics teachers in the relation between mathematics and the public. Perhaps we should go back to Félix Klein and give these actual or future teachers the necessary elements needed to round out their culture, especially concerning the history of their discipline. We should at least facilitate their access to documentary sources.

4. Other scientists, engineers and many professionnals in various fields require taylor-made expositions in a style that might be very different from our customary one. Demand needs to be stimulated and offers need to be facilitated.

5. Insofar as they are in contact with authors, publishers and institutions, the European Mathematical Society and national mathematical societies can be catalysts or even the driving force in the production of works of synthesis, reference works and high quality popularizing books. They can facilitate the work of authors through institutions or universities and see to it that such works are subsequently acknowledged.

6. It is also the responsibility of mathematical societies to improve communication between researchers, the potential authors of such works, and the media, who first of all trade in information. As an example, a campaign was launched in France by sending a letter to journalists and various institutions on the life of the mathematical community, the way departments work or the points of view of various social or intellectual figures more or less involved in mathematics. But this campaign needs refueling. Who else but researchers themselves can do it best?

   This is all the more true since journalists or other popularizers in the field are, we need to stress this again and again, rarely understood in their respective environments. Mathematicians who hate “wasting their time” do not imagine how hard it is for their interviewers to obtain a few lines or a few minutes on the air and evoke a subject whose austerity scares off their bosses. Perhaps mathematical societies could convey the message that, even if specialized research and rigor may lose a few points, public curiosity and intellectual stimulation may be aroused. They must also make it clear that a media product cannot make use of research language, but it can be provoking, thrilling, pleasant or even beautiful. From another aesthetic viewpoint: to talk about the subject rather than treating it precisely.

7. In several European countries, mathematical societies already take the initiative or even play an instrumental role in publishing the works of mathematicians. This is typically the case in Poland and Hungary. Other countries, like France, lag behind. In addition to publishing works, detailed notices—including obituaries—on the most important mathematicians should be developed. For Europe, the obituaries of the London Mathematical Society might provide a model.

8. Mathematicians must first of all learn to understand and communicate with each other by creating information networks on national and/or European levels.
This is one way of learning how to talk to non-mathematicians. This was already a key idea of the deceased President of the French Academy of Sciences, Jean Hamburger. The first “general public” for mathematicians is the mathematical community.

9. Still to be developed are the global aspects such as the relationship of mathematics with other disciplines concerning knowledge as well as comparative history and social and political stakes. Mathematicians must contribute to this undertaking which is in direct contact with the preoccupations of the public as well as of its decision-makers.

10. We need to think about the history of mathematics within the mathematical community. We also need to think about the reasons for the existence of research and its future directions, as well as the significance of mathematics within the intellectual hierarchies. Also of importance is the new role mathematics could play within a “mathematics and society” programme. In dealing with the relationships between mathematics and the general public, all aspects of the “mathematics and society” programme touched upon during the first European Congress of Mathematics must be taken into consideration and utilized and, in particular, the report written by Catherine Goldstein for the present volume.

References


