Mathematics in the French Educational System

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I. Secondary Education

A. The French school system

The French school system is highly centralized: structures, regulations, curricula and syllabi are laid down at the national level. National competitive exams are organized to recruit most secondary school teachers who are appointed by a central authority. The private sector represents just under 20%. Most of it is subsidized and controlled by the state. The percentage of children attending school is extremely high: 99% of the children go to school as soon as they are three years old and the proportion of 16 to 18 year olds has nearly doubled in the past 20 years. This effort has resulted in larger classes and a shortage of teachers, especially in mathematics.

B. The organization of secondary education

Secondary education follows nursery education (2 to 6 year olds) and primary education (6 to 11 year olds). There is no examination at the end of primary education. Secondary education is divided into two cycles and lasts 7 years.
— The first cycle concerns 11 to 15 year olds who are taught at a collège. It consists of the 6th, 5th, 4th and 3rd forms in ascending age order.
— The second cycle concerns 15 to 18 year olds who are taught at a lycée. It consists of the 2nd, 1st and Terminale forms.

All pupils in the first cycle receive the same education. At age 13, selection and guidance for general, technological or vocational streams takes place. This first cycle leads to an examination, the brevet, which has no influence on further studies or selection. The pupil is guided, often by his teachers, towards general, technological or vocational education, the last one being separate from the others. Compulsory schooling ends at the age of 16, following the end of the first form of the second cycle. At this point, general education is divided into diverse streams (A, B, S until 1992; L, ES, S after). Some of these streams are subdivided either at the age of 16 or 17. At the end of their secondary education at the age of 18, students take a national exam, the Baccalauréat. A vocational stream, which also leads to a Baccalauréat, has been developing rapidly in parallel with the general and technological streams.
C. Mathematics in secondary education

I. Mathematics at the collège

In the 6th and 5th forms, 3 hours are allocated to the teaching of mathematics in a global average of 25 hours a week. A fourth hour is however provided, either for all the pupils or for supportive or workshop activities. In the 4th and 3rd forms, pupils spend 4 hours a week on mathematics. New maths curricula were gradually introduced between 1986 and 1987, starting in the 6th form. The teaching is centered on tutorials in geometry, numerical analysis, data organization and processing and functions. Literal calculus and geometry are gradually introduced but are actually taught in the 4th form. An introduction to statistics is given and the use of pocket calculators is systematic.

II. Mathematics at the lycée

A complete pedagogical reorganization is currently under way, but will take effect only for the 2nd form at the beginning of 1992-1993. The 2nd form asserts itself as a guidance form for all. The notions studied are mainly notions which are likely to be useful to all pupils, especially with more practice in statistics. The weekly math timetable of 4 hours and 15 minutes includes a three-quarter hour period for modules. These modules are a novelty and aim to give the best possible individual help to each pupil in need, or in any case to help the pupil develop his or her abilities as much as possible. As these modules have no specific programme, they can give way to teaching initiatives while using as a basis the math content of the curriculum.

At present, pupils spend 2 hours a week in mathematics in the “literature and arts” and “literature and languages” sections, 5 hours in the “economics” and “literature-mathematics” section, while in the “science” section, they spend 6 hours a week in math in the 1st form and 6 and 9 hours respectively in the “math-biology” and “math-physics” Terminale sections.

After the excesses of the “modern mathematics” reform, new curricula were set up for the second cycle in 1981. New reorganized curricula were introduced in 1985 for the 5th forms and in 1986 for Terminales. To follow up the 1986-1990 curriculum reorganization in the first stage, new adjustments were necessary in the 2nd and 1st forms at the beginning of 1991 and 1992 respectively. The present mathematics curricula are not very ambitious concerning their contents. They no longer include linear algebra or studies of algebraic structures as such, and develop only a pragmatic and intuitive approach to the questions of limits and continuity. However, on the other hand, the basic geometry of configurations is quite developed with an emphasis on the “practical function of geometric applications.” In addition, the study of statistics is globally reinforced as well as that of probability theory. They are both taught in the 1st form in an elementary and pragmatic way, without being centered primarily on combinatorial analysis as it used to be.
D. Recent modifications

The Ministry of Education has recently launched several major reforms:
— the creation of a National Curriculum Council in charge of proposing the broad orientations of the education programmes;
— the creation of the Instituts Universitaires de Formation des Maîtres (IUFM). The IUFM are an amalgamation of the former institutions for the training of primary, general secondary, and vocational education teachers which were previously separate;
— the setting up of yearly national assessments of pupils at the ages of 8, 11 and now 15 (CE2, 6th and 2nd forms);
— a pedagogical reorganization of the lycées.

The last measure will concern 15 year old students in 1992 (2nd form), 16 year olds in 1993, and 17 year olds in 1994. As mentioned above, in the 2nd form modules have been introduced in four core subjects, one of which is mathematics. For each pupil this represents about 27 hours a year in each of the four subjects, during which he will work in a more flexible way and in a class reduced in size. This will be organized slightly differently in the following two forms, 1st and Terminale, where in each section a complex system of optional courses, offering either further study of a subject already taught or a new subject will be increasingly provided.

It is too early to pass a judgement on the impact of these measures.

II. Higher Education

A. The organization of higher education

The French higher education system is characterized by the coexistence of a multiplicity of institutions with different goals, structures and admission requirements. This diversity is reinforced on the level of the organization of studies and of the diplomas awarded.

One of the particularities of the French system is its heterogeneous structure in undergraduate higher education and the dualism of two types of graduate higher education institutions.

The *baccalauréat* is required and sufficient to undertake higher education studies. With the *baccalauréat*, students can choose between two systems: a system where entry is not selective at university and another one where entry is selective (school records or competitive exam) either at university in an *Institut Universitaire de Technologie* (IUT) or in a *Section de Techniciens Supérieurs* (STS) for short technological courses, or in a *Classe Préparatoire aux Grandes Ecoles* (CPGE) in a lycée or a *Grande Ecole* for graduate courses.

The student population in these four streams has considerably increased over the last ten years. The number of university students has increased by 3.3% a year
on average, with a 7.8% increase in selective streams, particularly in the STSs where it trebled between 1980 and 1990.

**Undergraduate higher education**

Undergraduate higher education essentially concerns industry and the tertiary sector. Training generally lasts two years (sometimes three), at the end of which a vocational diploma is awarded. The courses are generally provided by IUTs within universities and lead to a *Diplôme Universitaire de Technologie* (DUT), or within lycées in STSs, where they lead to a *Brevet de Technicien Supérieur* (BTS) and where the training is more specialized than in IUTs.

**Graduate higher education**

Graduate higher education is provided by universities and *Grandes Ecoles*, whose syllabi are organized differently.

**I. The universities**

Universities, which are state-run in their vast majority, have the monopoly of awarding diplomas. They have huge numbers of students and provide basic and practical courses augmented by research.

Universities are self-governing and multidisciplinary-oriented institutions. This autonomy also appears in the studies: universities determine their own curricula, which sometimes must be validated by the Ministry, the organization of their courses and how their diplomas are awarded.

Globally speaking, university studies can be divided into three cycles:

— a first cycle of multidisciplinary studies for students with the *baccalauréat*. It lasts two years and leads to an undergraduate degree, the *Diplôme d’Études Universitaires Générales* (DEUG) with possibly mentions of a specific discipline. The first-choice stream for mathematics is the “DEUG A SSM” which generally combines physics or computer sciences with mathematics. Other streams increasingly combine mathematics with social sciences (DEUG MASS), often with economics and statistics courses. The DEUG is essentially intended to serve as a qualifying cycle for the second university cycle;

— a second cycle of high-standards in-depth studies preparing students for professional responsibilities or for a third university cycle. The second cycle leads to the *Licence in one year*, the Maîtrise in two years and the *Magistère* in three years;

— a third cycle, lasting one or several years, which is highly specialized and oriented toward research. Two types of training are provided:

  — a one-year course with emphasis on vocational training and a compulsory training period in a firm, leading to a *Diplôme d’Études Supérieures Spécialisées* (DESS),
- training for and by research leading to a post-master qualification, a 
  Diplôme d’Etudes Approfondies (DEA) and then in principle in three 
  years time to a doctorate.

II. The Grandes Ecoles

The Grandes Ecoles are the second type of graduate higher education institu-

tions. They are specific to the French system.

The Grandes Ecoles, which have a smaller intake of students than the universi-

ties, provide high-standards training with vocational goals: training of teachers 
  and researchers, of the vast majority of engineers and business executives. Access 
  to the Grandes Ecoles involves a two or three years preparation in a CPGE within 
  some lycées and success at competitive exams. It is generally the choice of the 
  brightest students. Mathematics is relatively important in the scientific CPGEs. 
  There are two types of Grandes Ecoles: on the one hand, the specialist engineering 
  schools, which train engineers and civil and defense executives: Ecoles Polytech-
  nique, des Mines, des Ponts et Chaussées, Centrales, Navale, ENI, ENSAM, In-
  stituts Nationaux des Sciences Appliquées (INSAS = university colleges of applied 
  science), and on the other hand, colleges of education, such as the four Ecoles 
  Normales Supérieures (ENS), “the Ecole Normale Supérieure (Ulm-Sèvres) and 
  the ENS of Fontenay-Saint Cloud, Lyon and Cachan (ENSET), which train teach-
  ers and researchers. Studies in the Grandes Ecoles last three or four years. The 
  specialist engineering schools award an Engineer’s Diploma recognized by the 
  Council for Engineers Awards, while business schools award a diploma of higher 
  business studies.

The ENS prepare their students for national university diplomas and also to 
  take competitive examinations for the recruiting of teachers (CAPES, and espe-
  cially Agrégation).

B. Mathematics in higher education

I. At the university

About 50 of the 70 universities devote a fairly important share of their activity 
  to the teaching of mathematics. The student population in science, without taking 
  into account medicine, has risen very sharply to reach 215,000 in 1989-1990. This 
  situation has disrupted the teaching practices and methods in universities entirely. 
  This is even more so in mathematics since we are confronted with both a shortfall 
  in the recruiting of secondary school teachers and in the future replacement of 
  university professors, as well as with the need of taking charge of new teaching 
  duties and new, more vocational courses that were set up these last few years.

Combined with the changes in secondary education curricula, this rapid in-
  crease of the student population has prompted universities to alter the content 
  of their courses, especially DEUG courses, as well as their teaching methods, 
  particularly in the first cycle.
In addition, in order to attract more students to research, an active policy of incitement to research has been—and still is—developed for DEAs and doctoral courses. This policy is supported by granting students more facilities to carry on third cycle studies, especially through grants awarded by the Ministry of Research and Technology.

There has been a remarkable increase in the number of students who are awarded mathematics diplomas at all levels. However, the number of graduates is still definitely insufficient to meet the recruiting crisis of secondary education mathematics teachers. Similarly, the number of completed doctorates is insufficient to ensure that it will be possible to replace all higher education mathematics teachers when they retire. In addition, a genuine policy to make courses more vocational is being set up in universities. Mathematics has not escaped this change and several very vocationally-oriented mathematics courses have been designed: *Maîtrises de Sciences et Techniques* (MST), *Magistères, Diplômes d’Etudes Supérieures Spécialisées* (DESS) and a few mathematics engineering courses.

### 1.1 Mathematics in the first cycle of the university

It is in the DEUG A-SSM that mathematics has the most important position. But even in this section, the importance of mathematics and the type of training provided in it varies according to the streams (such as mathematics / physics, physics / mathematics, mathematics / physics / chemistry, mathematics / computer science, and so on).

**Organization of the courses.** Most of the first year is common to the various streams, or has a common core syllabus for about one semester. In nearly all cases, students have to pass an exam in order to proceed from the first to the second year. The DEUG diploma is awarded at the end of the second year after passing another examination. Generally speaking, continual assessment is given a great deal of weight: midterm examinations, written and practical work, “dissertations” or projects. In the first year students spend 9 to 10 hours a week in mathematics. In the second year, the number of hours varies notably according to the section considered, from about 12 hours a week in sections where mathematics prevails to 4 to 6 hours where physics and chemistry prevail, plus two to three hours a week in computer science at some point in either year.

**Teaching methods.** The courses consist largely of lectures. However, in some universities and in some sections there is a mixture of lectures and “problem classes and tutorials”. These initiatives are becoming increasingly difficult to maintain, due to the increase in the student population which has not always been accompanied by the appointment of more teachers. Some universities have also introduced practical study of mathematical applications using desktop computers.

**Mathematics curricula.** The curricula have been significantly altered over the past few years, in particular because they needed to be adapted to the new secondary
education curricula which are far less abstract than before. Basic notions of linear algebra are now taught in the first cycle of higher education. Probability theory is generally taught in the second year.

1.2 Mathematics in the second cycle of the university

1.2.1 The Licence (Bachelor’s degree)

The mathematics Licence involves a minimum of 500 hours, 300 of which are devoted to the following topics: differentiation, complex analysis, differential equations, Fourier analysis, integration and probability theory and numerical analysis. In addition to this, the mathematics Licence must include a course in computer science on the structure of computers, algorithmics and the learning of one programming language.

The mathematics Licence is not meant to be a final examination but should enable the student to acquire the necessary basic knowledge to carry on with more specialized studies like an M.Sc or vocational studies to prepare for a CAPES, which is a competitive exam for the recruiting of teachers.

While the number of students who obtained a mathematics degree had decreased in the 1980s, especially because of the development of computer science and of the low number of positions offered in competitive exams for the recruiting of teachers such as the CAPES and the agrégation, we now see a dramatic increase in the number of mathematics graduates: 2,600 in 1989-1990, that is to say, a 160% increase since 1986, whereas the number of computer science graduates has remained constant at about 1,300 since 1985.

1.2.2 The Maîtrise (Master’s degree).

The mathematics Maîtrise involves a minimum of 500 hours, including computer science training, and can be obtained in a wide range of options:

— The “pure mathematics” option must include 250 hours in the following topics: algebra, geometry and analysis;
— The “mathematics and basic science applications” option (MASF) must include a minimum of 250 hours in mechanical analysis, numerical analysis, probability theory and statistics and basic physics;
— The “mathematical engineering” option (MIM) must include a minimum of 300 hours in the following topics: numerical analysis, scientific computation, partial differential equations and distributions, probability theory and statistics, elements of mechanics and physics.

Again, the organization and content of Maîtrises degrees vary considerably. However, the pure mathematics Maîtrise primarily leads to teaching and basic research, the mathematics and basic science applications Maîtrise is designed for applications of mathematics to physics and mechanics and the mathematical engineering Maîtrise for scientific computing and industrial applications.
There also exist other Maîtrises where mathematics holds a dominant position, such as the “mathematical and computer science structures” Maîtrise, which is closer to computer science, the “discrete mathematics” Maîtrises with emphasis on logic, symbolic computation and cryptography and a few “science and technology” Maîtrises (MST).

The choice of a Maîtrise determines the choice of third cycle studies, either short, vocational (DESS), or long (DEA, doctorate). Moreover, with a Maîtrise a student can sit for the agrégation in mathematics.

I.3 Mathematics in the third cycle of university

The third university cycle is characterized by several diplomas at the level Baccalauréat + 5 years of studies, the Diplôme d’Etudes Approfondies (DEA), the Diplôme d’Etudes Supérieures Spécialisées (DESS) and the Magistère. After obtaining a DEA, a Magistère or a DESS, students can carry on their studies and prepare for a doctorate.

I.3.1 The DEA

The preparation of a DEA takes another year after a Maîtrise. It is the first year of doctoral studies at the end of which the student submits a doctoral thesis. The aim of a DEA is to consolidate in depth the knowledge acquired in a Maîtrise and to acquire more specialized knowledge. A DEA mainly consists of lectures (about 100 to 250 hours depending on the type of DEA), rarely combined with tutorials and practical work. Initiation to research is always a strong characteristic of the DEA, which provides training for research thanks to a training period. The DEA training period lasts several months and consists either in the study of recent research papers or in applied research work within a firm. Each training period is supervised by a professor and results in a written dissertation and an oral presentation. The final natural purpose of a DEA is to start preparing for a doctorate. However, because of their variety, DEAs also play the role of final examination which is appreciated in the work market.

I.3.2 The DESS

Unlike the DEA, the DESS is a final diploma of university studies. It provides highly specialized training and prepares students to directly take a job in mathematical engineering. It does not however entitle its holder to an engineer qualification, which is reserved to graduates from schools whose diploma has been validated by the “Council for academic awards”. Its structure is comparable to that of a DEA (lectures + training period) but more vocational. The nature of job opportunities implies a strong emphasis on numerical analysis, scientific computing, probability theory and statistics and even computer algebra. There now exist some university units that are entitled to award engineer qualifications, which proves that universities are becoming more vocational.
I.3.3 The Magistère

The Magistères are top-level university courses that combine basic knowledge, training periods, vocational qualification and initiation to research. The Magistères, which were introduced recently, are university diplomas validated by the Ministry of Education. Studies last 3 years and concern small groups of students selected by a competitive exam taking place after the DEUG or a classe préparatoire aux Grandes Ecoles. The possible openings are positions as engineers in a firm and/or researcher. At present, there exist in France six Magistères in mathematics, three of which are associated with the Ecoles Normales Supérieures. The three others are offered by large provincial universities that have important departments of mathematics, applied mathematics, and computer science. In addition to the traditional courses corresponding to national diplomas, students are offered further courses that are more vocationally oriented: applied mathematics, modeling and scientific computing. A common characteristic of all Magistères is the place reserved for personal projects and training periods in firms or laboratories.

I.3.4 The doctorate

The doctorate is the normal continuation of a DEA. It is generally prepared within a team or a laboratory of research in mathematics. The system of doctorates was recently changed in France: now there is only one doctorate. The preparation of a doctorate takes about three years. The doctorate must consist of original research work, conducted at university or in a firm, or in a mixed university-firm environment. A doctorate holder can then apply for a teacher-researcher or a senior lecturer position, or continue his career as a researcher or a mathematical engineer in a firm.

II. The CPGE-Grandes Ecoles stream

Except for a few institutions, especially the Instituts Nationaux des Sciences Appliquées (INSA), studies begin in the Classes Préparatoires aux Grandes Ecoles cycle in order to gain admission to the Grandes Ecoles, Ecoles Normales Supérieures (ENS) or engineering schools. The aim of this cycle is to intensively prepare the students for the competitive entrance exams to the Grandes Ecoles. This is achieved through hard and steady work, which also has a formative effect on the working methods of the students. The main CPGEs in which mathematics has a significant share are: science and technology, agronomy and biology, economics and business classes préparatoires.

The classes préparatoires to the science and technology Grandes Ecoles

The scientific classes préparatoires are the first cycle of engineering studies, but they also contribute a lot to the training of mathematics and physics researchers as well as secondary and higher education teachers. They prepare students in two
years for the competitive entrance exams to the various science and technology *Grandes Ecoles*, which generally train engineers as well as some researchers and teachers.

*The schooling*

It lasts two years. The first year, called *Mathématiques Supérieures*, cannot be repeated. At the end of the second year, called *Mathématiques Spéciales*, students take several competitive entrance exams to the schools they have chosen. Many of these exams are common to several schools. On the average, 90% of the students accepted in *Mathématiques Spéciales* manage to enter a school after one or two years.

*The courses*

Courses are mainly scientific: mathematics, physics and chemistry represent 60% to 80% of the syllabus. They also include technology, which can even be important in some sections, a foreign language and French. Students have a busy timetable: from 31 to 38 hours a week of lectures and “class tutorials”, to which practical computer science work, oral and written tests, not included in the timetable, must be added. The courses follow a national curriculum and the share allocated to mathematics depends on the section considered (from 12 to 14 hours a week).

*The competitive exams*

The 90 schools or so that belong to the *Conférence des Grandes Ecoles* organize competitive exams to recruit their students, most of them at the level of *Mathématiques Spéciales* and a few at the level of *Mathématiques Supérieures*. Apart from the three scientific *Ecoles Normales Supérieures* (Ulm-Sèvres-Lyon-Cachan), which essentially train teachers and researchers, these schools lead to an engineer’s diploma. The competitive exams generally consist of written and oral exams that take place at the end of the academic year. Mathematics, through a system of coefficients that is specific to each exam, plays a significant role in the results of the exam.

### III. Mathematics and Engineers

Short IUT and STS courses do not confer an engineering qualification. Some IUT sections have sections in which mathematics is important (DUT in statistics). In those two types of courses, mathematics is more likely to be a service discipline. The role of engineering schools is not to train mathematicians, but an engineer’s training must include substantial mathematical training. In the case of CPGE-engineering schools, training generally lasts three years after two (or three) years in a *classe préparatoire* during which students have studied a lot of math mainly in view of the competitive exams and in which mathematics seems to be more a
selection tool than a real culture. The mathematics taught in engineering schools is essentially applied mathematics and few schools provide a specialization in the field.

IV. Training and Recruiting Mathematics Teachers

A. Secondary education

For several years, the recruiting of teachers in science, especially in mathematics, has been in a state of crisis. This crisis is due to several main factors: retirements of teachers, increase in the student population in the lycées and university first cycles, low number of graduates and a job that has become less attractive on the job market.

This crisis was made worse by the development of post-baccalauréat education, especially that of computer science. In addition, the number of positions offered through the national mathematics CAPES and agrégation competitive exams had dropped sharply between 1975 and 1980. It has been increased again in the past few years. This policy resulted in a sharp drop in the number of candidates. However, their numbers are now comparable to those of the mid-seventies. In the same period, the number of mathematics graduates increased considerably (from 1,052 in 1986 to 2,899 in 1991).

To curb this crisis, the Ministry of Education sponsored reflection groups and made a certain number of decisions concerning the recruitment competitive exams and the training of future secondary education teachers: more positions at the CAPES/agrégation recruitment exams, revaluation of teachers salaries, grants and allowances for students who want to embark on teaching careers. The global answer is however the setting up of the Instituts Universitaires de Formation des Maîtres (IUFM) in all regions since the beginning of the academic year 1991-1992.

The IUFMs replace previous structures: Ecoles Normales d’Instituteurs (ENI), Ecoles Nationales Normales d’Apprentissage (ENNA), which trained the teachers of vocational secondary schools, Centres de Formation des Personnels de l’Enseignement Technique (CFPET) and the Centres Pédagogiques Régionaux (CPR) which were designed to prepare those who had passed the theoretical CAPES exam for the practical teaching part of the same exam. IUFM students are recruited among Licence graduates, on the basis of school records and after an interview. IUFM training lasts two years. For future collège and lycée teachers, the first year is devoted to the preparation of the CAPES competitive exam, a preparation which necessarily includes practical teaching experience. The second year, which is more vocational, is centered on a training period when students are in charge of a class. During that year their status is that of “trainee” teacher.
B. Higher education

To accommodate the foreseeable need for educators and professors caused by the increase in student population and retirements, the Ministry of Education has significantly increased the number of fellowships awarded since 1989. The same year, Centres d'Initiation à l'Enseignement Supérieur (CIES) were set up. These centres have been designed for post-graduate tutors and post-graduate-Normalien-tutors recruited among students who have been awarded research fellowships. A tutor's position is not meant as a pre-recruiting of teacher-researcher, but gives the student an extra grant (an incentive) for higher education training. For the first time this year, the first batch of such “tutors” took the 1992 higher education recruitment competitive exams.

V. Some Aspects of Research into the Teaching of Mathematics

Research into the teaching of mathematics began within university teams and Instituts de Recherche sur l'Enseignement des Mathématiques (IREM) in the early seventies and gained momentum in the wake of the modern mathematics reform. Gradually, the didactics of mathematics has developed around mathematics teaching but also around teachers' training. From now on, with the setting up of the new IUFM institutions, teacher training will systematically include a didactic course and the CAPES exam a more vocational test taking this new aspect into account.

Note: A longer version of this report with more detailed figures is available from the author.