The Role of Mathematics in Educational Policies in Germany

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Before presenting any detailed description concerning the Federal Republic of Germany, we must mention the drastic changes that took place in Germany from 1989–1990. After decades of separation between East- and West-Germany, Germany became politically united. As a consequence, German authorities are still struggling to develop a uniform, or at least compatible, educational structure for the whole nation. Education continues to be under regional control with the now 16 \textit{Ländere} as the decision making bodies. Presenting a national picture implies overlooking important regional differences in West Germany and emerging structures in East Germany.

I. Secondary and Higher Mathematics Education in Germany

1.1 Structure of general education

For secondary education in Germany—especially West Germany: the new \textit{Ländere} tend to imitate West German structures—there are two most important borderlines. The first one separates primary from secondary education after 4 years of schooling (for the majority of \textit{Ländere}. A few place this borderline between grades 6 and 7). At this point, the pupils normally decide which type of secondary school they wish to attend. This decision already implies the choice students have at the second borderline. After 10 years of schooling, at the age of 16, full-time compulsory education is over. Students then either attend a continuation of general education (in most cases a Gymnasium, a grammar school, about 30\% of the population) with the objective of obtaining an entrance qualification for the university (higher education), or the majority of 16 to 17-year olds enter vocational education and/or the work force with additional part-time college attendance until the age of 18 (see Figure I next page). Special schools exist for the physically and/or mentally handicapped, but will not be described in the following.

Table 1 (adapted from Weidig) offers some statistical data on the respective importance of the schools and their development over the years.
Table 1. Students in Secondary Schools.

All percentages only apply to the 11 Länder forming the Federal Republic of Germany before 1990.

<table>
<thead>
<tr>
<th>School (without special schools)</th>
<th>1931</th>
<th>1966</th>
<th>1988</th>
<th>1988 (from state to state)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Volksschule/Hauptschule</td>
<td>78%</td>
<td>62%</td>
<td>36%</td>
<td>16%-45%</td>
</tr>
<tr>
<td>Realschule</td>
<td>6%</td>
<td>18%</td>
<td>29%</td>
<td>21%-35%</td>
</tr>
<tr>
<td>Gymnasium</td>
<td>16%</td>
<td>20%</td>
<td>30%</td>
<td>27%-35%</td>
</tr>
<tr>
<td>Gesamtschule (integrated type)</td>
<td>5%</td>
<td>1%-28%</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Figure 1. Educational system of Germany
I.2 Mathematics education in secondary general education

All students have to attend mathematics courses till the end of the lower secondary level. At the age of 13, the student normally has four lessons in mathematics per week. In most Länder, mathematics is compulsory until the end of Gymnasium and most of the technical/vocational colleges have compulsory mathematics until the final examination. For the last two years of Gymnasium, two different mathematics courses are offered: About 75% of the students take a basic course in mathematics (normally 3 lessons per week), while about a quarter of Gymnasium students take an extended mathematics course (5 or 6 lessons per week). For them, mathematics is a subject with a written examination for the “Abitur” (the final examination in Gymnasium). There are large differences between male and female students concerning the choice of mathematics courses. At the university level, 18.3% of the male students take sciences and/or mathematics, while 13.8% of female students take sciences and/or mathematics.

Mathematics is taught in all schools and colleges as a separate subject, with the rare exception of some minor and highly specialised types of vocational colleges in grades 11 and higher (for details see Blum and Strasser). Nevertheless, it is somewhat difficult to briefly describe the mathematics curricula because of two bones of contention:

1) There are 16 Länder with 16 different curricula; one Land cannot relate to the another; each one fights for the regional right to make decisions affecting the particular curricula.

2) With the tripartition of the educational system at the secondary-1-level, for most of the Länder, we find three different curricula for mathematics. Normally, the syllabus for Hauptschule would be more oriented toward arithmetic and practical aspects of the application of mathematics, while the syllabus for Gymnasium would be oriented toward preparing for university entrance, especially in mathematics. With the role of Realschule differing from Land to Land (and some new Länder do not even have Realschule), the mathematics syllabus of Realschule is in some cases more oriented to Hauptschule, in others more to Gymnasium.

Table 2 (adapted from Weidig) presents an overview on the subjects taught, identifying topics usually taught in Gymnasium. Syllabi in Hauptschule/Realschule normally present a reduced table of contents.

It is in the teaching of geometry that the differences from Land to Land are the most important. In some Länder, transformation geometry was reduced to the study of the symmetry of some figures in grades 5 or 6 (e.g. Nordrhein-Westfalen and Rheinland-Pfalz). In others, e.g. Hessen and Niedersachsen, all of transformation geometry is included in the syllabi. In Table 2, probability/statistics is mentioned only for grades 11 to 13. Nonetheless, Hessen, Niedersachsen and Nordrhein-Westfalen prescribe this subject also in Hauptschule (grades 7 to 10).
### Table 2. Subjects taught in mathematics in secondary education
(especially Gymnasium)

<table>
<thead>
<tr>
<th>Grade</th>
<th>Subjects</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grades 5 and 6 (lower secondary education)</td>
<td>use of variables, simple equations, transforming of magnitudes, area of rectangles, volume of blocks, fundamental geometric forms, digits and number systems, elementary number theory (prime numbers, g.c.d., l.c.m.), fractions and decimal fractions</td>
</tr>
<tr>
<td>Grades 7 and 8 (lower secondary education)</td>
<td>relations and functions, commercial arithmetic (proportions, percentages), congruence transformations, congruent triangles, quadrilaterals, angle measurement and related theorems, area of polygons, volume of prisms, transformation of terms, sentential forms, linear equations, algebraic structures, integers and rational numbers</td>
</tr>
<tr>
<td>Grades 9 and 10 (lower secondary education)</td>
<td>the real numbers, quadratic functions and equations, similarity, theorems on right triangles and circles, powers, power and exponential functions, area of circle, volume of pyramid, cylinder, cone, sphere, trigonometry</td>
</tr>
<tr>
<td>Grades 11, 12 and 13 (Gymnasium only, upper secondary education)</td>
<td>calculus, vector spaces, linear algebra, analytic geometry, geometric transformations, conics, probability and statistics</td>
</tr>
</tbody>
</table>

On the whole, there may be about 10% of the curriculum devoted to probability and statistics at the lower secondary level, while at the upper secondary level of Gymnasium, stochastics may occupy 20% of the curriculum. Algorithmics is not a separate discipline in mathematics, whereas computer science is offered as a separate course in Gymnasium (upper secondary level, 3 lessons per week) and as an extension course in most schools of the lower secondary level. Because of the organisational and curricular problems, mathematics teaching has nearly no explicit links with other school and college disciplines. However, some teachers try to connect their math teaching with problems from other disciplines.
As for the teaching method, most of the time is devoted to an activity characterized by questions from the teacher followed by answers from the students (fragend-entwickelnder Unterricht, see Maier and Voigt). Lecture-type lessons (similar to those in the university) seem to be rare, below 10%. More time is reserved for exercises and problem-solving by individuals or groups of students. It is impossible to give an indication on the importance of projects, reports and so on, because the relevance of this type of activity differs widely even from school to school. A certain focus on project work can be identified in the pedagogy of comprehensive schools.

As for evaluation and tests, most of the tasks assigned seem to be of the same problem type as is common in homework. Apart from a large number of tasks involving variations of parameters in problems already solved in the classroom (mere exercises), word problems seem to play a role in evaluating and grading the students. Some teachers also use multiple choice tests to evaluate student progress (this is not widespread) and with higher grades, some reasoning may be required in tests. Real problem solving seems to be restricted to project work.

1.3 Mathematics teaching in university and higher education

As each individual Land has the right to decide on cultural and educational matters, we have several types of higher education: traditional universities, technological universities (Technische Hochschule), comprehensive universities (Gesamthochschule) and correspondence courses (also at university level). In the following, we will speak of “university” whenever we want to describe features of higher education that are common to all these types of institutions. All universities are autonomous in teaching and research. For the training of future teachers, each Land has its own regulations concerning the studies and examinations.

All universities (of whichever type) have a department of mathematics, which in most cases is part of a faculty of (natural) sciences, mathematics and computer sciences. Teaching mathematics at university level is comparable to math teaching all over the world at the same level. Students who study mathematics usually have to take calculus, linear algebra and applied mathematics during the first 4 semesters and pass a first examination (Vordiplom). For the final examination, the Diplom, they have to major in at least two mathematical subjects (e.g. topology, algebra, number theory, geometry, applied mathematics, statistics and so on), write a paper and pass the mostly oral final examination. Each university has its own curriculum and regulations for the diploma. Nevertheless, a diploma awarded by any German university will be accepted everywhere in Germany.

If the candidate passes his (her) diploma very successfully, he or she is invited to write a Ph.D thesis and to do research in a special field. After being awarded the title of Doktor of mathematics/natural sciences (which is comparable to a Ph.D) he/she normally finds a position as assistant-professor and prepares for the habilitation, an outstanding mathematical research work. After passing this examination, the person gets the so-called venia legendi, the privilege of teaching
and doing research mathematics at university level. The title then is Privat-Dozent. One now can apply for a full professorship position everywhere.

*Estimated time spent at the university:*

- Math diploma: 6 to 7 years;
- Doktor (Ph.D): an additional 2 to 3 years;
- Habilitation: an additional 2 to 4 years.

Total duration of studies: 10 to 14 years!

We have a hiring crisis for university teachers caused by the restrictive policy of the Länder governments. A young university teacher (Privat-Dozent) has little chance of being appointed in his or her profession at a university.

**II. Training Teachers: Mathematics as a Service Discipline**

Mathematics plays a role in many disciplines and studies undertaken at Fachhochschul- and university-level. Consequently, it is taught to a large number of students not enrolled in mathematics as a service discipline. It is rather difficult to obtain a clear picture of how this type of mathematics is taught, who teaches it, and what role it actually plays in the studies of the students. In fact, service mathematics may be taught either by staff from the faculty of mathematics or by (non-)specialists from the discipline mathematics is serving. In smaller universities, there may even be lectures for mathematics students (teachers and/or diploma) which are also attended by students studying the subject as a service discipline.

Depending on the subject to be served, it may or may not be compulsory to take mathematics for a specific diploma, while in some areas students may be able to avoid studying mathematics altogether. Usually, at least students in physics, chemistry, biology and engineering will have to take some mathematics. For these disciplines, the lectures usually cover calculus, linear algebra and (for physics and engineering) differential equations and numerical analysis. Students in social sciences are forced to attend some lectures in probability/statistics. It is known, but not researched in depth, that mathematics is one of the subjects that increases dropout rates in a variety of disciplines.

**III. Training Mathematics Teachers**

In relationship to the structure of the educational system in Germany, we have different kinds of teachers for general education. In this paper, we will not comment in detail on the training of teachers for vocational/technical colleges.

**III.1 Primary and secondary-I teachers**

For the primary level (Grundschule) and parts of the lower secondary level (Hauptschule, Realschule), we have pedagogical academies (Pädagogische Hochschulen) which were intended to train teachers in an elementary fashion separated from the university. Some Länder integrated the Pädagogische Hochschulen
into universities, while in other Länder the Pädagogische Hochschulen still exist. Pedagogical high schools or faculties of education are heavily overcrowded.

Every future Grundschule, Hauptschule or Realschule teacher has to attend some lectures in elementary mathematics and has to pass an examination in mathematics (elementary algebra, calculus, geometry, number theory, stochastics and so on). The reason for this is that every primary level teacher should be able to teach mathematics.

The training of these teachers proceeds in two phases:

1st phase (theoretical):
Studies at the Pädagogische Hochschule or at the university in two subjects (e.g. mathematics and ...), with only a few hours of practical teaching in schools during this phase of study. The 1st examination includes the writing of a paper in one subject and an examination by an examining board set up by the Land government.
Estimated duration of studies: 6–8 semesters.

2nd phase (practical):
Work as trainee teacher with part-time teaching under the guidance of experts in various types of schools, additional lectures and seminars at a teacher training institution (Seminar) on didactics, methodology of teaching and psychology. The 2nd examination includes the writing of a second paper, in most cases a report about a teaching unit, the teaching of two test-lessons and an oral examination at the Seminar.
Time spent at the Seminar: 2 years.
The total training time for Grundschule, Hauptschule and Realschule teachers is about 5 years.

There is a hiring crisis: many students pass both examinations but only a few of them are employed by the governments.

III.2 Lower and upper secondary level: grammar school (gymnasium)

1st phase (theoretical):
The future teacher has to study two subjects (e.g. mathematics and ...) and pedagogy including didactics at a university or comprehensive university. The government of each Land determines regulations. Each university has its own curriculum. It is impossible to give a detailed overview.
Normally, the 1st phase is divided into two blocks:
“basic studies” in calculus, linear algebra, applied mathematics, algebra, complex analysis, stochastics for teachers and so on, with an examination at the end of about 2 years (4 semesters).
“main studies” in at least two different fields of mathematics: algebra, geometry, topology, probability, number theory, applied mathematics and so on, for 3 to 4 years (6 to 8 semesters).
The final examination at university level (Erstes Staatsexamen) includes a paper (theoretical work on...) in one of the candidate’s subjects (e.g. mathematics or ...), two written examinations and a one hour oral examination in each subject plus half an hour in pedagogy and didactics. It is controlled by an examining board of the Länd.

2nd phase (practical):

The teacher training institutions (Seminars) are totally separated from the universities (and from the Seminars for primary school teachers). They are institutions under government control with experienced teachers of grammar schools as trainers. Under the guidance of these expert teachers (Fachleiter), the teacher trainees are expected to learn how to teach (e.g. mathematics). In addition to practical work, they receive further didactical, pedagogical and psychological instruction. This phase ends with an examination (Zweites Staatsexamen), organised by the expert teachers (Fachleiter) and a delegate from the government. The examination is made up of a test of the teaching qualifications in each subject, a written paper (often a report about a teaching unit) in one subject and an oral examination before a committee.

The total time necessary to train a teacher for Gymnasium is about 9 years!

There is a hiring crisis again. Although many schools sadly lack teachers, especially in mathematics, there are restrictions on hiring.

IV. Research into Mathematics Teaching

Didactics of mathematics

In Germany, a lot of research on the teaching of mathematics is done in the field called “didactics of mathematics”. Nearly every university has some staff specialising in this area, which to a large extent is organised in a research association called Gesellschaft für Didaktik der Mathematik (GDM)—Society for the Didactics of Mathematics. The GDM has its own research journal (Journal für Mathematikdidaktik—JMD). There is one national institute for research on didactics called the Institut für Didaktik der Mathematik (IDM) in Bielefeld, which also houses a documentation centre and library on the teaching and learning of mathematics.

In order to give you an idea of the research in the field, we adapt Burscheid, Struve and Walther and present a non-exhaustive list of research-foci in Germany:
— Elementarisation of mathematics (Stoffdidaktik) analyses pieces of mathematics in order to develop teaching units on the topic. Detailed studies on the role of proof in mathematics are undertaken.
— Applications of mathematics are studied and transformed into teaching units to enrich the teaching of mathematics and boost motivation.
— Methodological aspects of mathematics education are studied with a special
attention to the construction of mathematical knowledge in the classroom.
— Historical and philosophical investigations are used to better understand the epistemological dimensions of mathematics education and optimise mathematics teaching.

V. Concluding Remarks

Rather bluntly, we present four conclusions on the role of mathematics in German education:
— In Germany, the educational system seems to be not as comprehensive but more selective than in other European countries. Comprehensive schools only play a minor role in Germany; partly because of the unification of Germany in 1989, partly due to the growing number of students in Gymnasium, there is a political and educational debate on the future structure of the German educational system ("comprehensive" vs. "tripartite", the role of Gymnasium).
— In Germany, we have a hiring crisis for mathematics teachers and researchers partly because of financial pressure on education as a whole.
— From a historical perspective, the curriculum in schools proved to be rather stable. This is illustrated by the fading influence of the "modern math movement" on long-term curriculum development.
— Judging from the topics taught, mathematics seems to play a major role in the selection of the work force, contradicting the widespread legitimizing of school mathematics by the importance of mathematics for the competence of the work force.

References