a significant fashion, this moment corresponds also to that in which Mersenne and others launched a call, via their correspondence networks, for exchanges which would help constitute a European mathematical community. Though it was not the first such, it would be the first to unite mathematicians at this social level. Among those involved figure a number of ‘heros’ of modern science: Galileo, Wallis, Descartes and Huyghens among others.

The Perenniality of Programmes

based on the presentation of Henk Bos

University of Utrecht, Utrecht (Netherlands)

Descartes is often mentioned as the founding father of modern European mathematics. But there were some others at that time who thought about mathematics in as visionary and programmatic a way as Descartes. One of these was Viète and there are strong similarities between Descartes’ programme and the one Viète formulated some forty years earlier. This view is somewhat unorthodox because the current view stresses the differences rather than the similarities in the achievements of these two mathematicians. According to this view, Viète’s great achievement was to have introduced into algebra the use of letters both for unknowns and for indeterminates; but he bogged down in unwieldy notation and superfluous adherence to the homogeneity of equations. The current view about Descartes is that he introduced “our” notation in algebra, disregarded homogeneity and, most importantly, applied algebra to the geometry of curves, for which he created coordinate geometry and the method of studying curves by means of their equations.

These views are not completely wrong if you look at what lasted of Viète’s and Descartes’ achievements, but they are strongly misleading if you want to know what they really did and especially why they did it.

Thus, there is a tension between what is now considered important in the work of Viète and Descartes and what they themselves felt to be important. That tension has to do with realities and myths, to which we shall return, but first their programmes.

Both Viète and Descartes wanted to develop algebra as a tool for problem solving according to the following schema:

- translation of problem into algebra → equation → (algebraic solution of equation, if possible) → solution by retranslation of answer (the equation or its algebraic solution) back into original context

The context decided what would count as a solution; when the problem is about numbers the solution is a number arrived at through calculation, but when
the problem is geometrical, a solution is a configuration effected by means of a
construction. The latter case requires the geometrical construction of the roots
of equations. And since such roots often cannot be constructed with ruler and
compass alone, decisions have to be made about what will count as acceptable
means over and beyond the ruler and compass.

The problems which Viète’s and Descartes’ general algebraic method were
meant to solve are not related to modern ideas of algebraic structures; they aimed
their new methods at problems largely inherited from classical mathematics. Thus,
there was a rich tradition of geometrical problem solving, dealing with such ques-
tions as:

- the elementary Euclidean constructions,
- mean proportionals (including duplication of the cube),
- angular sections (dividing an angle in a given number of parts or following a
given proportion)
- addition of similar areas or volumes,
- placing a given section between two given curves and directed towards a given
point (the neusis problem),
- division problems (dividing an area or volume by a line or plane according to a
given ratio)
- triangle problems (constructing a triangle from three given elements)
- varia (e.g., find the normal to a parabola from a given external point)

These were distinguished, in a classification recorded by Pappus, into plane
(solvable by ruler and compass), solid (non-plane but solvable by the intersection
of conics) and line-like problems (all the rest).

Descartes worked out the programme further than Viète had done. Indeed
his main achievements in his Géométrie of 1637 was, formulated in seventeenth
century terms, that he gave a complete theory of geometrical construction of roots
of equations. To do so he explored the use of higher order curves as a means
of construction. He thus instituted a research programme that lasted for about
a century, then died, and is now completely forgotten. Coordinate geometry,
curves and equations also figured in the Géométrie, but primarily in the service
of the geometrical construction of roots of equations. They were taken up later,
enthusiastically, but almost despite the programme of the book. What Descartes
really aimed at was soon lost; what people took over were the techniques, not the
motivation. In that sense one may see Descartes as a tragic hero.

One could of course dismiss an interest in such abandoned programmes as
those of Descartes and Viète as irrelevant for real mathematics and, therefore, as
uninteresting history. As someone recently wrote about a study which endeavored
to make a serious analysis of Weierstrass’ concept of number:
"Why did Weierstrass cling to such a clumsy and badly presented method
as late as 1886, when much more satisfactory ones were available?"

Clearly such dismissals hinge on value judgements about past events — and
that is where myths are born!

Obviously one might think that this interest is valuable as well, for instance,
because the story of Descartes' and Viète's programmes shows how mathemati-
cians struggle to interpret the aims of their activity and to decide what should count
as exact or proper mathematics and what should not.

But in defending this interest in past mathematics which is no longer con-
sidered alive, one is up against a strong ideology which is at the root of much
confusion, misunderstanding and even contempt for the history of mathematics.

That ideology of mathematics starts from the conviction that the only value in
mathematics resides in what is in its top level, in brilliant and lasting mathematical
results or proofs or arguments; and that consequently, the only relevant selection
criteria for thinking about mathematics are the brilliance and permanence of the
results. Applied to history, that conviction leads to a teleological view of the
development of mathematics; the essence (if not the logic) of the development
of mathematics lies in how the brilliant results we know now as those essentially
valuable in mathematics were arrived at.

For instance, in a recent review of a historical study, the reviewer complained
that what he called "the essential and therefore the perennial" had not been made
clear and proceeded to make it clear — in mid-twentieth century symbols and
notation.

With respect to an historical understanding of mathematics, this ideology, and
the resulting teleological view of the history, is inadequate, if not harmful. It is so
because it is intellectually unsatisfactory, unserious about people and mathematics,
and not contributive to self-understanding (it also produces boring history). By
concentrating on the "great" results, one gets a depersonalized history; the people,
the mathematicians, are no longer the active agents but rather the mathematical
ideas or concepts which "emerge" in the course of history. In this view Descartes
is graced with the distinction of having helped coordinate geometry to "unfold".
But others may be blamed for "holding up" this development. Such a misfortune
happened even to Newton, about whom a standard history of the calculus has this
to say:

"Had Newton devoted more of his time to clarifying the elements of thought
in his demonstration by ultimate ratios, the calculus might have been es-
ablished upon the concept of the derivative a century before the time of
Cauchy."

Such a point of view takes seriously neither the people involved nor their
mathematics. The mathematicians of the past worked at their mathematics, not