Practices for identifying, supporting and developing mathematical giftedness in school children: The scene of Israel

Boris Koichu, the Technion, Israel, on behalf of EMS Committee of Education

From the history

Since its establishment in 1948, Israel has become the home for many scientists who were educated in Europe. Thus, it is not surprising that the first special activities aimed at identifying and supporting mathematically gifted school students were essentially aligned with a classic model created in Hungary by the end of the 19th century (e.g., Koichu & Andžāns, 2009). Namely, the identification was organized by means of high-level mathematics competitions, and the support – by means of activities around a high-quality journal.

The oldest in Israel mathematics competition (since 1960) is named after an inspiring mathematics teacher of the Reali school in Haifa and a later head of the Mathematics Section of the Technion (the Department of Mathematics since 1950), Professor Yirmiyahu Grossman (1884-1965). The first multi-stage nationwide mathematics competition was organized in 1968 by Professor Joseph Gillis (1911-1993), a prominent mathematician and one of the founders of the Faculty of Mathematics at the Weizmann Institute of Science. Professor Gillis was also among the editors of the first mathematics periodicals for school students and amateurs and the first leader of the Israeli team for the International Mathematics Olympiad (IMO), in which Israel takes part from 1979.

The periodicals are issued, under different names (the current name is Net-Gar), from 1960 and until today. The competitions – they are known as the Grossman Olympics and the Gillis Olympics – still maintains the high level of standards (see Figure 1 for examples of problems offered recently at the second stage of the Gillis Olympics). The Gillis Olympics, which begins from the test sent to all interested schools and continues with the competition conducted at the Davidson Institute of Science Education at the Weizmann Institute of Science, serves, in particular, as a tool for choosing an Israeli team for the IMO.

Problem 5 (2012). There is a Cartesian coordinate system in a plane. A point in a plane is called integer if its two coordinates are integers. A triangle having the following properties is given: its vertices are integer points, there are no additional integer points on its sides, and there are exactly four integer points within the triangle. Should these four points be necessary on a straight line?


Problem 4 (2014). Let $m, n$ and $k$ be positive integers so that $n^m$ is divided by $m^n$ and $m^k$ is divided by $k^m$. (A) Prove that $n^k$ is divided by $k^m$. (B) Find the aforementioned numbers $m, n$ and $k$ so that they would be all different and greater than 1.

Figure 1: Examples of problems from the Gillis Olympics (for $10^{th}$, $11^{th}$ and $12^{th}$ grade students)

Additionally, exceptionally gifted school students, who could not get appropriate support within the school system, could get individual response from prominent mathematicians working in the Israeli universities. Professor Abraham Berman from the Technion warmly recalls (Berman &
Leder, 2009) how encouraging for him as a high-school student has been his correspondence with one of the leading experts in axiomatic set theory and the first dean of the Faculty of Mathematics at the Hebrew University of Jerusalem, Professor Abraham Fraenkel. Professor Berman, who himself is known for his work in the field of non-negative matrices, also recalls stories of outstanding school students approaching him for advice and worthwhile opportunities to study advanced mathematics. For instance, Maria Chudnovsky was a school student when Abraham Berman invited her to participate in a graduate course on graphs and matrices at the Technion. Maria Chudnovsky is now a professor at Princeton University; she is known for proving, along with Robertson, Seymour and Thomas, The Strong Perfect Graph Conjecture in 2002.

In sum, it is safe to say that nurturing mathematical giftedness in a small number of exceptionally gifted school students has been driven in Israel by enthusiasm and dedication of a few school teachers and university professors. Their honorable work, however, is not a substitute to special programs for the gifted and the promising, in which hundreds or thousands of school students take part.

Nevo and Rachmel (2009) point out two equally important reasons for which a society should invest resources into establishing special programs for gifted children: for the sake of children who must be rescued from an unchallenging environment of their “normal” class, and for the welfare of the society that expects to benefit from future achievements of the gifted. Both reasons fully apply to Israel, in which the human capital is its primary quality resource for the country's development (Nevo, 2004).

The Ministry of Education established the organizational framework that caters to the needs of gifted students in 1973. The societal awareness of the need to nurture scientific talent from the childhood grew up especially during two last decades. It is worthwhile noting that the awareness grew up with the arrival and accommodation of a big wave of immigration from the countries comprising the former USSR at the beginning of the 1990s. This wave brought to Israel many excellent scientists and teachers who bore the traditions of mathematics schools and classes of the former USSR. Having said this, it should be noted that the contemporary Israeli system for identifying, supporting and developing mathematical giftedness in school students essentially differs from the Eastern European’s and the former USSR’s systems.

**Guiding Principles**

In a recent analysis of the ways of nurturing students with high mathematics potential, Leikin and Berman (2016) indicate that gifted education in Israel relies on the following positions: the *equity principle* (refers to equal opportunities for students with different needs), the *diversity principle* (refers to the need for a diverse student body), and the *challenge principle* (refers to the need for challenging tasks and goals).
principle (refers to the diversity of fields, in which human talent can be manifested), dynamic perspective (acknowledges that cultivating human talents requires designing unique learning environments, distinct study tracks, appropriate teachers and curricula), and holistic approach (acknowledges the amplitude of instructional approaches for promoting the amplitude of abilities and skills, such as creativity, intellectual integrity and the ability and desire to continually learn).

These positions reflect and stipulate the Israeli scene where various in-school programs and out-of-school activities for mathematically promising students are conducted in different forms and formats. Some of them run as special projects of the Ministry of Education, some are led by Israeli universities, and some are conducted as initiatives of particular schools or public associations.

Current Practices

Practices for elementary school students

As a rule, the Israeli first graders are enrolled in elementary school (Grades 1-6) by the place of residence. Accordingly, mathematics (as well as the rest of the elementary school subjects) is studied in heterogeneous classes. The teachers are encouraged to use complementary instructional materials for those students who show signs of being insufficiently challenged by the standard materials.

In Grades 2 or 3, all Israeli children take an examination for determining eligibility for acceptance to two special governmental programs. The examination consists of the tasks that test general cognitive ability (so called IQ-type of tasks) and are not related to the school’s curriculum. About a year after the multi-stage acceptance process, which is essentially based on this examination\(^2\), about 1% of children are enrolled, upon their agreement and the agreement of their parents, to special classes for the gifted that operate in selected elementary and secondary schools (Grades 3-12) around the country. The classes are relatively small (up to 26 students), and the curriculum in these classes expands the regular national curriculum with respect to both depth and breadth. Top 5% are eligible for acceptance to a weekly enrichment day program that operates in 52 regional centers for elementary school students (Grades 3-6). The students leave their regular school for one day a week in order to study in the center a variety of scientific topics, from medicine to mathematics. The students are expected to catch up with the missed in the school material by their own, and this usually works. In brief, there is no special governmental education for mathematically gifted elementary school students, but there are special programs for the generally gifted.

Local educational initiatives focused on mathematics and science also exist. Interested parents can relatively easy find for their children an after-school mathematics club, forum or circle operated by a university (e.g., the program Kidumatica - The Youth Mathematics Forum is operated by the Ben Gurion University of Negev, see Amit & Neria, 2008) or a public association (e.g., MOFET\(^3\)).

Recently, the Math-by-mail program for mathematically promising or curious students (Grades 3-9) gained a good response all over the country. The program is offered by the Davidson

\(^2\) See [http://cms.education.gov.il/EducationCMS/Units/Gifted/English/Gifted_Acceptance_test/Acceptance_process.htm](http://cms.education.gov.il/EducationCMS/Units/Gifted/English/Gifted_Acceptance_test/Acceptance_process.htm).

Institute for Science Education and combines online activities, weekly meetings in schools and an annual conference⁴.

**Practices for junior high school students**

The number of opportunities for mathematically promising students increases in junior high school (Grade 7-9). There is a one-level official mathematics curriculum for junior high school, but a two-level streaming is a widespread practice. Many schools conduct an internal examination during the 7th grade or use regular grades and teacher recommendations in order to split the students into A-stream and B-stream classes in mathematics lessons, usually starting from the 8th grade. Additionally, the Ministry of Education, in cooperation with the Technion and the Hebrew University of Jerusalem, conducts a nationwide program, which gives an equal number of extra hours to low-achieving and high-achieving in mathematics students. Thus, a bright junior high school student would normally study in an A-stream class and attends one-two additional enrichment lessons a week in his or her school.

There is also a variety of out-of-school frameworks for junior high school students. A good example is RANGE – The Interdisciplinary Center for Research and Advancement of Giftedness and Excellence at the University of Haifa. The RANGE Center conducts research in the field of giftedness and simultaneously involves in research activities highly motivated junior high school students, who attend the Center for its various academic programs in the field of mathematics, science, art and neuroscience⁵.

Additionally, there exists a system of special mathematics classes of different levels of inclusiveness, in which mathematics is studied 7-9 hours a week, compared to regular 5 hours a week. Examples include: the Technion classes, TELEM (Technion Lessons in Mathematics) classes, MOFET classes (the curriculum, teacher training and didactical support is provided by a mathematics department of the MOFET association), Eshel-Ha-Nasi school’s mathematics class in Beer Sheva (this special class is being initiated by a school teacher, Dr. Peter Samovol, who is an expert in mentoring student research projects in mathematics).

Advanced mathematics tracks exist in special schools for generally gifted students, such as the Reali school in Haifa, The Israel Arts and Sciences Academy in Jerusalem, and the Leyada (an abbreviation in Hebrew for "near the university"), which is closely connected to the Hebrew University of Jerusalem. One of the graduates of the Leyada was Professor Elon Lindenstrauss, a 2010 Fields Medalist.

The most prestigious and selective program is *The Future Scientists and Inventors Program* initiated by President Shimon Peres⁶. It is a four-year program that begins in Grade 9. The program is managed by the Rashi Foundation and the Ministry of Education, in cooperation with Tel Aviv University, the Technion, Ben-Gurion University, the Hebrew University and Tel Hai Academic College. The program is for exceptionally gifted in mathematics students (about a quarter of the top percentile of an age cohort) who show strong motivation along with social commitment; the students are chosen in a four-stage acceptance process. The participants in the program attend one of the above mentioned universities two days a week⁷ in order to take

---

⁵ See [http://range.edu.haifa.ac.il/about-range-center](http://range.edu.haifa.ac.il/about-range-center).
⁷ Similarly to the participants of *Weekly Enrichment Day Program*, the participants in *The Future Scientists and Inventors Program* are supposed to catch up the missed in school material by their own.
academic courses, hear the lectures by distinguished professors and industry leaders and experience laboratory work. In sum, an exceptionally gifted 9th grader in Israel would normally combine the study in one of the aforementioned types of special mathematics classes with the participation in *The Future Scientists and Inventors Program*.

**Practices for senior high school students**

Depending on the level of achievements in junior high school, Israeli 10th graders are enrolled in mathematics study at one of three levels: low, regular or high. The high-level mathematics curriculum in senior high school (Grades 10-12) differs from the regular one in terms of depth of study of the prescribed topics\(^8\), but the most salient difference is related to the complexity of the problems involved. Leikin and Berman (2016) note however that the high-level mathematics curriculum is not exclusively devoted to the mathematically gifted. Currently, 8% of senior high school students study mathematics at the high level, and the fact that this percentage is relatively small concerns the Ministry of Education as well as some segments of the Israeli society.

Needless to say, there are quite many students who need more advanced opportunities to study mathematics than even the high-level curriculum can provide. As indicated above, some of the mentioned programs and activities begin in elementary and junior high school and continues up to the senior high school. Not yet mentioned programs for senior high school students include: integration of school students in university courses and mathematics research camps.

Many Israeli programs for mathematically promising and gifted students (e.g., Bar Ilan University Acceleration Program, the MOFET classes, the *ETGAR* (the acronym for *challenge*) program at the University of Haifa, the Tel Aviv University Program for Locating and Promoting Outstanding School Students in Mathematics) include acceleration. The participants in these programs are given a chance to pass the matriculation exams in Grades 10 or 11 and begin studying academic courses towards B.Sc. degree. In some cases talented students can be accepted to the universities as regular students without a matriculation certificate. There are exceptional cases when outstanding students finish their B.Sc. study (and even M.Sc. study) along with their high-school study.

In-school and out-of-school programs aimed at engaging students in research are in the mainstream of Israeli education for the gifted. Besides *The Future Scientists and Inventors Program* and RANGE program, opportunities to do mathematics research when in high school are provided, for instance, by *The Noam Center for Talented Youth* in cooperation with the Department of Mathematics of the Technion. The Center organizes each summer a two-week Number Theory Camp for about 20 talented high school students of grades 9-12. In the Camp, the students learn the Number Theory basics and cope, with the help of the Technion students and faculty, with challenging problems requiring exploration. The Center also runs an international summer research program for high school students (Grades 10-12) named SciTech. SciTech attracts gifted students from many countries. The 3-4 week program combines doing scientific research in several fields including mathematics (see Berman, Goldberg & Koichu, 2005, for examples of student projects) with cultural and social activities.

In sum, Israeli senior high school students exhibiting interest, promise, giftedness or exceptional giftedness in mathematics can find programs suitting their needs.

---

\(^8\) The topics are: algebra, probability, plane and solid geometry, trigonometry, vectors in plane and in space, calculus, complex numbers, functions (polynomials, rational function, exponential and logarithmic functions
Mathematics competitions

The main kinds of mathematics competitions (see Koichu & Andžāns, 2009, for a review) are represented in Israel. The aforementioned Gillis Olympics and Grossman Olympics for students of grades 10-12 are high-level competitions, the winners of which are granted scholarships and can register for study in the Israeli universities without additional requirements.

The main mathematics competition for junior high school student is Zuta Olympiad (Mini Olympics). As a rule, students who are interested in mathematics competitions begin their way to the Israeli team for the IMO from participating in Zuta Olympiad. The winners of this competition are invited to training camps at the Weizmann Institute along with the winners of the Gillis Olympics. It is in place to mention that Israel does not make the short lists at the IMOs; the country’s best achievements were the 11th place in 2000 and the 13th place in 2013.

There are also several mathematics competitions “for all”, including the Open Competitions (since 1989; see Movshovitz-Hadar, 2008) and Kangaroo (since 2013). However, the mathematics competition movement in Israel is less massive than in some other countries.

Trends

Mathematics education, including education for those school students who show mathematical promise, is on the societal agenda. The main special programs for mathematically promising and gifted students are developed by enthusiastic professionals (i.e., mathematicians, mathematics educators and mathematics education researchers) and jointly operated by the Ministry of Education and the universities.

Preparation of sufficient number of teachers who can teach the high-level mathematics curriculum and work in the variety of special programs is the bottleneck of the system. An emerging trend is a greater involvement of the public sector and IT industry. For instance, the Trump Foundation is involved in many projects aimed at enhancing mathematics teacher training. Additionally, obtaining an additional B.Sc. degree in mathematics, science or technology education becomes more and more popular among middle-age IT specialists who look for opportunities to begin the second career or looks for ways to contribute to the educational system while continuing working in industry.

An additional trend is integrating distance learning into regular school timetables. The Virtual High School, a joint initiative of the Ministry of Education, the Center for Educational Technology and the Trump Foundation, has recently become reality for hundreds of students who study in peripheral schools, in which there are not enough students for opening classes of high-level mathematics. The best teachers reach these students by means of an interactive digital platform.

Finally, there is a trend of incorporating mathematics education for exceptionally gifted students in regular school system. Since 2015, a new high-school curriculum containing four (instead of three) levels of studying mathematics is launched. The new advanced high level is for about 1% of high school students who aspire to become professional mathematicians, whereas the high level remains for those who strive for careers in IT industry (currently 8% of high school

9 See http://www.trump.org.il/en
10 An example is the Technion VIEWS program. See http://www.focus.technion.ac.il/Jan15/education_story2.asp
students, and the Ministry’s goal is to reach 15% in 2020). It is planned that the Virtual High School will operate also at the new advanced high level of studying mathematics.

The variety of the presented programs and activities should not overshadow a somewhat strange fact: there are no special schools for mathematically gifted students in Israel. The future will show if there is the need for establishing such schools.

References


