What are the Reciprocal Expectations between Teacher and Students?
Solid Findings in Mathematics Education on Didactical Contract

Education Committee of the EMS

Episodes from school

Four examples of episodes from school are used in order to address a relevant solid finding in mathematics education. Nevertheless, the idea and importance of reciprocal expectations between teacher and students may be illustrated by means of many other episodes.

Episode 1

This episode is known in the community of didacticians under the name "the captain's age". At the end of the 1970s, the researchers from IREM (Institut de Recherche sur l'Enseignement des Mathématiques) Grenoble proposed, without really manifesting reasons, the following assignment to primary school students: "There are 26 sheep and 10 goats on the boat. How old is the captain?" Seventy-six of 97 students calculated the captain's age by combining the given numbers by some operation like addition or subtraction.

Various versions of the captain's age problem were used in many countries (see e.g. Verschaffel, Greer & de Corte, 2000). Similar behaviour of students was observed in most cases. It is governed by their belief that the data in the problem assignment are to be used in the calculations and these calculations give the required answer. Most students do not try to make sense of the assignment and trust the teacher that the problem is correctly assigned.

Episode 2

This story took place in a class of 9–10 year old students. The teacher taught the following algorithm facilitating calculation of the difference between two numbers:

$$328 \xrightarrow{+3} 331 \xrightarrow{+50} 381$$
$$\underline{-47} \xrightarrow{+3} \underline{-50} \xrightarrow{+50} \underline{-100}$$
$$281 \qquad 281$$

Several weeks later, the students were assigned the following task.

How would you carry out the following calculations?

a) $875 - 379 =$ _____

b) $964 - 853 =$ _____

c) $999 - 111 =$ _____

Most students (16 out of 19) applied the algorithm they were taught in all the three exercises including the third one:

$$999 - 111 = 1008 - 120 = 1088 - 200 = 888$$

The students' reaction is governed more by what they suppose the teacher expects from them than by the nature of the question. Most of them prefer to show their ability to use the taught algorithm than to calculate the difference between 999 and 111 directly.

Episode 3

This story takes place in a class of 13–14 year old students. The following equation from homework is written on the whiteboard:

$$5\frac{2}{3}x - \frac{3}{2} = 4\frac{1}{6}x + \frac{1}{2}$$

What happened is that several students replaced the mixed numbers $5\frac{2}{3}, 4\frac{1}{6}$ by the expressions $5 \times \frac{2}{3}, 4 \times \frac{1}{6}$ and then carried on using the correct solving procedures. This mistake was not anticipated by the teacher and the differences between the two cases became the topic of the ensuing whole class discussion.

The main characteristic of this teacher's work is that she keeps referring back to reasoning about rules that were taught and validated a long time ago. Her students trust that this reference is to the former knowledge that is useful when solving the assigned problem(s) and rely on it. However, when assigning the homework, the teacher did not make any link to her students' former knowledge of mixed numbers. As a consequence several students worked with them incorrectly.

Episode 4

This story takes place in a class of 15–16 year old students. In the test, the students are asked to solve the following problem. Find $x \in \mathbb{R}$ such that: a) $\sin x = \pi/3$; b) $\cos x = \pi/2$.

Only 25 % of the students give the correct answer to a) and 29 % to b).

The students act according to their belief that the teacher always presents tasks that have a solution. For example, explicitly citing from a discussion with one student: "It is strange that an exercise could have no solution."

In some mathematics classes these or similar situations happen quite often, while in other classes they do not happen or are very rare. If the source of these types of situation was the mathematical content, they should occur in all classes. Research in mathematics education has confirmed that the source is more or less in implicit

rules that regulate the relations between the teacher and their students and are class-specific. This set of implicit rules is called the *didactical contract.*

Genesis of the notion of didactical contract and its relevance at present

The concept of didactical contract (DC) was proposed by Brousseau in France at the end of the 1970s with the objective of explaining specific failures that can be found only in mathematics (Brousseau called them "elective failures"[1]). As we can see from the above examples, students often answer to comply with what they think is expected from them by the teacher rather than to cope with the assigned situation. The DC is a theoretical construct invented in order to deal with this phenomenon. It is certainly one of the most fundamental theoretical constructs in the didactics of mathematics both on a French and an international scale.[2]

The most cited definition of DC is Brousseau's (1980, p. 127): the DC corresponds to "the set of the teacher behaviours (specific to the taught knowledge) expected by the student and the set of the student behaviours expected by the teacher".

The DC is the set of reciprocal obligations and "sanctions" which

- each partner in the didactical situation imposes or believes to have imposed with respect to the knowledge in question, explicitly or implicitly, on the other;
- or are imposed, or believed by each partner to have been imposed on them with respect to the knowledge in question.

The DC is the result of an often implicit "negotiation" of the mode of establishing the relationships for a student or group of students, a certain educational environment and an educational system. It can be considered that the obligations of the teacher with respect to the society which has delegated to them their didactical legitimacy are also a determining part of the "didactical contract".

The DC is not a real contract because it is not explicit, nor freely consented to; moreover, neither the conditions in which it is broken nor the penalty for doing so can be given in advance because their didactical nature, the important part of it, depends on knowledge as yet unknown to the students.

Furthermore the DC is often untenable. It presents the teachers with a genuinely paradoxical injunction: everything that they do in order to produce in the learners the behaviour they want tends toward diminishing the students' uncertainty and hence toward depriving them of the conditions necessary for the comprehension and

the learning of the notion aimed at. If the teacher says or indicates what they want the student to do, they can only obtain it as the execution of an order and not by means of the exercise of the students' knowledge and judgment (this is one of several didactical paradoxes brought about by the DC). But the student is also confronted with a paradoxical injunction: the student is aware that the teacher knows the correct solving procedure and answer; hence, according to the DC, the teacher will teach them the solutions and the answers; they do not establish them for themselves and thus do not engage the necessary (mathematical) knowledge and cannot appropriate it. Wanting to learn thus involves the student refusing the DC in order to take charge of the problem in an autonomous way. Learning thus results not from the smooth functioning of the DC but from breaking it and making adjustments. When there is a rupture (failure of the student or the teacher), the partners behave as if they had had a contract with each other.

The DC is not an illness of the didactical relation; it shows that learning mathematics consists not only of memorising algorithms and knowing definitions. It is the object of the theory of didactical situations (Brousseau, 1978, 1997) to study situations that allow the learner to learn to do mathematics and not only to memorise it. For example, if students practise a number of exercises for addition of two numbers and the teacher inserts a subtraction exercise, students who only memorise mathematics will continue adding the numbers.

The DC is structurally analogous to the well known social contract of J.-J. Rousseau: the social contract allows us to understand theoretically the conditions of existence of relationships between an individual and a group, without postulating that this social pact occurred, in a certain way, among social agents; everything happens as if this apparent accordance had been contracted some time ago. It equally concerns subjects of all didactical situations (students and teachers).

Another paradox implied by the DC is the one identified in the theory of situations under the name 'the paradox of devolution': the teacher has to talk to students who de facto cannot understand because they must learn and, as Brousseau says, "if the teacher says what it is that she wants, she can no longer obtain it" (Brousseau, 1997, p. 41). An example of such a situation is published in Novotná (2009). One of the questions discussed is the following: are students able to recognise, clarify and explain similarities and differences between problems and are they able to recognise various problems related to one mathematical model in different conditions? All the activities are organised in such a way that they involve spontaneous emergence of the notion of mathematical model; the notion is not taught explicitly by the teacher.

At every moment of the lesson the teacher might be assuming that as a consequence of teaching, their students have or have not learned something. Analogically, the student may think that they really understand what the teacher is trying to teach. But when one or the other tries to verify this, a system of expectations evolves, with whose help all the involved parties make decisions on the

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1 Students with a specific failure are those "who have deficiencies in acquisition, learning difficulties or lack of liking, shown in the domain of mathematics but who do sufficiently well in other disciplines" (Brousseau, 1978).
2 Guy Brousseau was the first to be awarded the Felix Klein medal from ICMI, not least because he proposed the notion of DC to the community of mathematics education researchers.

extent of concord between what was observed and what was expected (called *hypothetical contract* by Brousseau). Even in the case that the observation matches the expectation, nothing can guarantee that this concord testifies that the learning itself is really in accord with what the teacher expected.

The following example illustrates the aforementioned phenomenon. When working with multiplication tables, the teacher assigns the task '*Fill in numbers into the boxes*':

$$♠♠ \quad ♠♠ \quad ♠♠$$
$$\square \times \square = \square$$

The answer expected by the teacher is $\boxed{3} \times \boxed{2} = \boxed{6}$.

It is not possible to decide unequivocally merely on the basis of this answer (unless it is supplemented by additional comments) whether the student really grasped multiplication of natural numbers. The student might have used a simple process based on the following algorithm: I know that the teacher requests that I should fill in the number of ellipses in the first box and the number of spades in one ellipse in the second box and I know that the total number of spades is to be written in the third box. Therefore, what students do is count the number of spades rather than multiplying 3×2.

Authorship

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Additional information

A slightly expanded version of this article with a more complete list of references may be found on the web at http://www.euro-math-soc.eu/comm-education2.html.

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MSC 2010 in SKOS – the Transition of the MSC to the Semantic Web

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Abstract: The most comprehensive subject classification scheme in mathematics is the MSC (Mathematics Subject Classification), based originally on an AMS classification scheme, a standard more than 20 years ago. Zentralblatt MATH and Mathematical Reviews jointly developed the first MSC 1990. The current version is MSC 2010 following another 10-year update. With the trend from printed to digital publications, it turns out that the original structure of the MSC is not capable of expressing what we want to convey in the semantics of electronic publications. SKOS (Simple Knowledge Organization Scheme) is an open standard vocabulary intended for the modelling of thesauri and classification schemes for the Web. The SKOS representation of the MSC 2010 will be a start to making it more usable for electronic publishing and integrated into the Semantic Web. This article outlines why and how this has been done.

1. Mathematics classification systems

Classification schemes are intellectually created conceptualisations for organising particular subjects. For searching mathematical publications by subject, ordering them in catalogues and bookshelves or describing the context of a publication in a systematic way, indexing of these publications according to subject is a basic requirement. In the past, several schemes have been developed for this pur-