# Solid Findings on Students' Attitudes to Mathematics

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## Episodes from ordinary school life

### *Episode 1:*

Alicia (7th grade) has to find the perimeter of a rectangle whose length is 12 cm and whose width is 8 cm. She multiplies 12 by 8. The teacher says: "Why did you multiply? You have to find the perimeter, not the area…!" And Alicia says: "Shall I divide?"

### *Episode 2:*

Nicholas (11th grade) has to solve the inequality:

$$-7x^2 < \sqrt{7}$$

He multiplies both sides by –1/7, obtaining:

$$x^2 > -\frac{\sqrt{7}}{7}$$

Then he multiplies by 7 and transposes the right member to the left side:

$$7x^2 + \sqrt{7} > 0$$

At this point he stops and refuses to continue in spite of the teacher's encouragement.

Most people – teachers and mathematicians too – will probably describe Alicia's and Nicholas' behaviours as effects of a 'negative' or even 'wrong' attitude toward mathematics, which leads the students to answer randomly or to refuse answering. But what does 'negative attitude' really mean? And how can this diagnosis help mathematics educators in planning remedial actions? In other words, how can mathematics teachers modify a student's negative attitude toward mathematics?

## Negative / positive attitude towards mathematics

The construct *attitude* finds its origin in social psychology (Allport, 1935), in connection with the problem of predicting an individual's behaviour in contexts that involve choices based on simple preferences like buying goods or voting. In these studies, attitude is generally described as a predisposition to respond to a certain object either in a *positive* or in a *negative* way. In mathematics education, early studies about attitude had already appeared in the second half of the 20th century, moved by the belief that something called 'attitude' plays a crucial role in learning mathematics (Neale, 1969): they tried to highlight a causal relationship between (positive) attitude toward mathematics and school mathematics achievement.

The characterisation of attitude in these early studies is that typical of the social sciences, seeing attitude toward mathematics as the emotional disposition toward the discipline, thus identifying a positive/negative attitude toward mathematics as a positive/negative emotional disposition ('I like/dislike mathematics'). In this early period, some important results have been obtained, in particular about the relationship between attitude towards mathematics and the choice of mathematical courses (Aiken, 1970) and about gender differences (Fennema & Sherman, 1977). But, in actual fact, research on attitude failed in reaching its main goal: a clear correlation between attitude towards mathematics and mathematical achievement does not emerge (Aiken, 1970; Ma & Kishor, 1997). For example, McLeod (1992) refers to data from the Second International Mathematics Study, which indicate that Japanese students had a greater dislike for mathematics than students in other countries, even though Japanese achievement was very high.

The acknowledgment of this failure and the attempts of interpreting it contributed to point out the need for a theoretical debate about research on attitude. Researchers highlight as critical points in previous research the lack of a suitable definition of positive/negative attitude and the inappropriateness of the instruments used to measure it (obviously also 'success' and therefore 'achievement' in mathematics can be defined and measured in different ways).

As regards the definition, characterising attitude simply as the emotional disposition toward mathematics can be useful in dealing with issues such as the choice of mathematics courses or the comparison between different groups of individuals but it seems inadequate in dealing with complex issues such as success in mathematics, which involves the decisions made by an individual during problem solving activity. In this case, a 'positive attitude' toward mathematics cannot be reduced to a positive emotional disposition but should be linked with 'positive' beliefs about the discipline, i.e. with an epistemologically correct view of it. A suitable characterisation is made by Richard Skemp (1976), who identifies an *instrumental* vision of mathematics, according to which mathematics is a discipline made of fixed rules without reasons to be memorised and applied, as opposed to a *relational* vision of mathematics, characterised by a focus on the processes and their relations rather than on products.

In fact, in the early period, the instruments used to measure attitude, even when attitude was simply seen as the emotional disposition towards mathematics, were mainly questionnaires constituted of items that refer not only to emotions ('I like mathematics') but also to beliefs about mathematics ('Mathematics is useful') and to behaviours ('I always do my homework in maths'). In some way therefore they seem to take into account the vision of mathematics. But most choices appear to be questionable about the evaluation of positive/negative (that results in the assignment of a score). For example, with regard to beliefs about mathematics, agreement with the items 'Mathematics is useful' or 'Mathematics is easy', which are most used in questionnaires on attitudes, is considered 'positive' but many mathematicians might question this choice.

These critical points also influence the approach to the crucial issue of promoting a positive attitude toward mathematics. Is the goal of developing a positive emotional disposition toward mathematics, if this disposition is not associated to a 'positive' view of the discipline, a significant goal in mathematics education? Considering only the emotional aspects has a great risk: it can lead teachers to avoid complex tasks in order to promote a positive emotional disposition.

## A new period for research on attitude

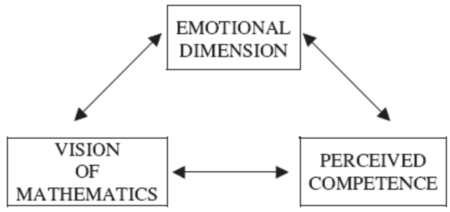
A new period for research on attitude toward mathematics arose from the debate about the critical issues described above, characterised by the need for a theoretical framework and for new methods of inquiry. This need involves research on affect, a new field in mathematics education that, in addition to attitude, also includes constructs such as emotions and beliefs (see McLeod, 1992; Törner, 2013). Recent research on affect takes into account a radical critique that at the end of the 1980s arose within the social sciences and later involved mathematics education: the limits of a normative approach, i.e. the attempt to explain behaviour through measurements or general rules based on a cause-effect scheme. The awareness of the high complexity of human behaviour gradually led to the affirmation of an alternative paradigm: the interpretive one, aimed at understanding – rather than explaining through universal laws – an individual's actions.

Recent studies therefore abandon questionnaires in favour of narratives (essays, diaries, interviews) and also of the observation of behaviour in natural settings or in structured situations. In particular, the use of narratives makes it possible to take into account the subjectivity of an individual's attitude towards mathematics.

An example of a study that uses narratives to describe attitude toward mathematics is that carried out by Di Martino and Zan (2010), who collected more than 1800 autobiographical essays with the title '*Maths and me: my relationship with maths up to now*' written by students of all grade levels, trying to identify how students describe their relationship with mathematics. From this study emerges that when students describe their own relationship to mathematics, nearly all of them refer to one or more of these three dimensions:

- Emotions.
- Vision of mathematics.
- Perceived competence.

These dimensions and their mutual relationships therefore characterise a student's relationship with mathematics, suggesting a Three-dimensional Model for Attitude (TMA):




**The TMA model for attitude (Di Martino & Zan, 2010)**

Interestingly enough, the teacher emerges as a crucial mediating factor with respect to these three dimensions and it is the most recurrent factor linked to changes in a student's attitude toward mathematics.

The multidimensionality highlighted in the model suggests the inadequacy of the positive/negative dichotomy for attitude that refers only to the emotional dimension. In particular, the model suggests considering an attitude as negative when at least one of the three dimensions is negative (identifying, according to Skemp, as negative an *instrumental* vision of mathematics, and as positive a *relational* one). In this way, it is possible to outline different *profiles* of negative attitude towards mathematics.

With this model in mind, let's go back to Alicia and Nicholas, protagonists of the episodes described. Actually, we know more about them than the crude description made above: we have further information obtained through the use of observational tools, aimed at better interpreting their actions.

Alicia has written in her autobiographical essay: "At elementary school I was not a genius in mathematics, so in the third class I realized that I was not good and therefore closed my head, saying that mathematics was not for me."

Nicholas was involved in an interaction with a young researcher, who intervenes in front of his block, saying: "Why don't you try to solve this inequality by reasoning, instead of remembering the correct way to follow?"

Nicholas answers: "Mathematics is done by fixed and precise rules that have to be respected and applied: you can not invent anything. To solve problems you have to follow them, and in this moment I don't remember the rules to solve inequalities."

This further information allows us to understand the apparently irrational behaviour of Alicia and Nicholas. Alicia's words reveal that she is convinced of not being able to do mathematics, highlighting her low perceived competence in that field. Nicholas seems to view mathematics as a discipline made of fixed rules without reasons, to be memorised and applied; in other words, he seems to have an instrumental vision of mathematics instead of a relational one.

Even with this information, we continue to see the two students' behaviours as a consequence of a negative attitude towards mathematics. But the theory on attitude developed in mathematics education gives us new instruments to interpret these kinds of behaviour and then to intervene effectively. Alicia's *profile* of negative attitude appears to be different from that of Nicholas, thus requiring different didactical actions: in the case of Alicia, the negative component is her low perceived competence in mathematics and the primary goal of the teacher should be to convince her that she 'can' do mathematics; in the case of Nicholas, the negative component seems to be his instrumental vision of mathematics and the goal of remedial action should be to overcome this vision, in favour of a relational one.

### Some conclusions

Although the debate about some critical issues in research on attitude towards mathematics still continues, this research has produced some solid findings and we will highlight them here.

The most important such solid finding is, in our opinion, that non-cognitive factors have a crucial role in learning mathematics; this 'belief' has been the starting point of research on attitude, has motivated interest for the construct of attitude and has become a finding, since research has highlighted the deep interaction between cognition and affect in the context of mathematics.

But in its evolution, research on attitude has also contributed to highlight another general and significant issue in mathematics education: the need to adapt constructs and tools borrowed from other fields in order to face problems that are specific to mathematics education. This position characterises mathematics education as a discipline problem-led rather than method-led; in other words, research in mathematics education is led by problems, which influence the search for suitable methods, and not by the methods available, which impose what kind of problem can be dealt with.

### Authorship

Even though certain authors have taken the lead in each article of this series, all publications in the series are published by the Education Committee of the European Mathematical Society. The committee members are Tommy Dreyfus, Ghislaine Gueudet, Bernard Hodgson, Celia Hoyles, Konrad Krainer, Mogens Niss, Juha Oikonnen, Núria Planas, Despina Potari, Alexei Sossinsky, Ewa Swoboda, Günter Törner, Lieven Verschaffel and Rosetta Zan.

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# ERME Column

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### CERME8, Side-Manavgat Antalya, Turkey
### 6–10 February 2013
### http://www.cerme8.metu.edu.tr/

The 8th Congress of the European Society for Research in Mathematics Education (CERME8) was held at the Starlight Convention Center, Thalasso & Spa Hotel, in Manavgat-Side, Antalya, Turkey, 6–10 February 2013, chaired by Prof. Dr. Behiye Ubuz (Local Organiser Chair) and Prof. Dr. Maria Alessandra Mariotti (International Programme Committee Chair).

At CERME8, considering ERME policy, we wanted to allow groups of researchers in a particular scientific area to really work together on their area of research, with sufficient time to get to know each other, to share and discuss their research and to engage in deep scholarly debate. At the same time, we wanted to support the scientific development of young researchers fostering their active participation in our research community. Therefore, at CERME8, participants spent most of the time in discussion and debate within the thematic Working Groups (WGs) over seven working sessions of 90 minutes each. The leaders team organised the peer review process among the members of the groups according significantly devolved and distributed responsibility in criticising but also supporting the elaboration of the